Topology in condensed matter physics
Exercise sheet 6

University of Hamburg
Thore Posske

6.1 10-fold way

Given a Hamiltonian $\mathcal{H}$ that has no unitary symmetry, a time reversal operator $\mathcal{T}$ is an antiunitary operator that commutes with $\mathcal{H}$, a particle-hole operator $\mathcal{P}$ is an antiunitary operator that anticommutes with $\mathcal{H}$ and a chiral symmetry $\mathcal{C}$ is a unitary operator that anticommutes with $\mathcal{H}$. These operators, if they exist, square to $\mathcal{T}^2, \mathcal{P}^2 \in \{-1, 1\}$, and $\mathcal{C}^2 = 1$.

Symmetries

$$T : \text{Antiunitary} \qquad [T, H]_- = 0 \tag{1}$$

$$P : \text{Antiunitary} \qquad [P, H]_+ = 0 \tag{2}$$

$$C : \text{Unitary} \qquad [C, H]_+ = 0 \tag{3}$$

Helpful properties

$$[A, BC]_\pm = [A, B]_- C + B[A, C]_\pm = [A, B]_\pm C \mp B[A, C]_-, \tag{4}$$

with $[A, B]_\pm = A \pm B$.

1. (2 points) Show that $\mathcal{C} = \mathcal{T}\mathcal{P}$, $\mathcal{T} = \mathcal{C}\mathcal{P}$, $\mathcal{P} = \mathcal{C}\mathcal{T}$ (understood up to possible scalar phases).

Kramer's degeneracy

Consider a Hamiltonian H with a time reversal symmetry T that squares to -1, i.e., an antiunitary symmetry T with $[H, T] = 0$ and $T^2 = -1$.

1. (2 points) Show that for each eigenvector $|\phi\rangle$ of H, the state $T|\psi\rangle$ is an eigenstate of H that is orthogonal to $|\psi\rangle$.

End