Topology in condensed matter physics

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consent for video recording

Organization, content, motivation, basic topological definitions

Organization Language: English

Lecture times: Wednesday 10^{00} am - 11^{30} am s.t.

Exercises: Wednesday 11^{45} am - 13^{30} p.m.

[starting next week]

↳ warm-up exercise online

[No lecture on Dec 25, Jan 1]

→ video recorded lectures, course assistent: Anna Junker

webpage: www.posske.de → lectures → TICMS 2024/25

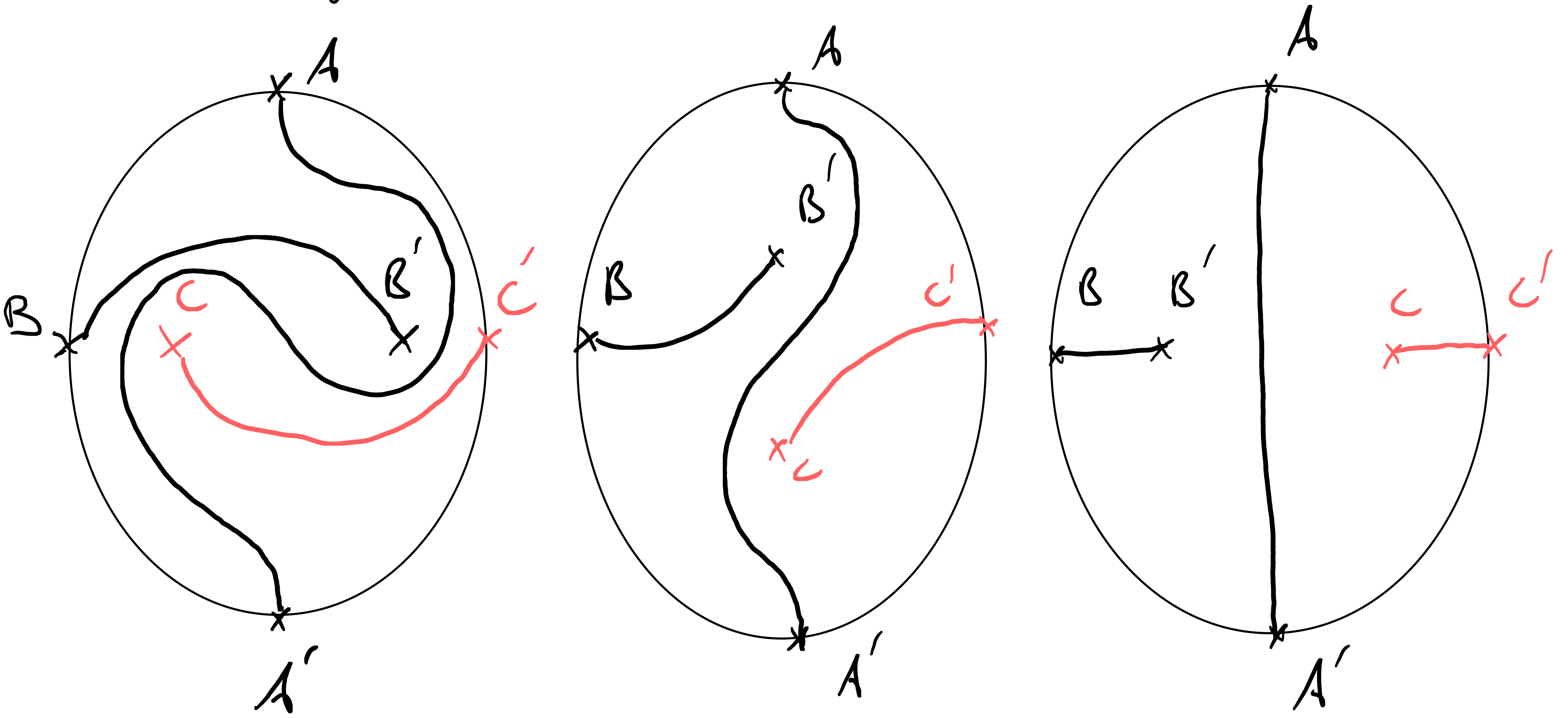
user: SelectedTopics pw: Haldane1951

Voted on:

! **Exam**: written/orals, date voted on next week

! **times of lectures**:
- overlap with QFT lecture
- additionally possible times?
- break between lecture & exercises
- exchange time slots of lectures & exercises?

Wake up brain & motivation



Connect AA', BB' & CC' by nonintersecting, continuous curves inside the disk.
Is that possible? If not, prove your result.

Edward Witten

"Topology is the property of something that does not change when you bend or stretch it as long as you do not break anything [or glue things together]"

Motivated from above example

understand topology $\longleftrightarrow$ draw topology

Topology in physics

Electrodynamics
↳ Gauss, Stokes

General relativity
↳ black holes
↳ topology of space-time

Focus ! condensed matter physics
↳ quantum Hall effects (normal, anomalous, fractional, spin)
↳ anyons & topological quantum computing
...

Content of the course

knowledge poll + most selected additional topics

Topology in condensed matter systems

I Topology

- set theoretic definition
- open & closed sets
- metric spaces
- special constructions
  - ↳ product spaces
  - ↳ quotient spaces
  - ↳ fibre bundles !

[· differential topology]

II Noninteracting electronic systems

- gapped phases & superconductors
  - ↳ definition of top. phase
  - ↳ fundamental symmetries
  - ↳ Cartan-Altland-Zirnbauer classification
    The tenfold way
  - ↳ topological invariants
  - ↳ examples: Kitaev chain

[· ungapped phases]

III Adiabatic time evolution & anyons

- Kato's approach to adiabatic propagation
- topological quantum computing
- Indiscernible particles
  - ↳ configuration space
  - ↳ bosons & fermions
  - ↳ anyons in 2D (& 1D)

I Topology

Throughout the course, we take a naïve approach on sets as collection of elements. A *family* is a collection of sets.

topology Let X be a set & τ a family of subsets with the properties

1) The whole set & the empty set is in τ

$X \in \tau$, $\phi \in \tau$ $\quad [\phi \equiv \{\}]$

2) Unions of elements of τ are in τ.

For $\mathcal{U} \subseteq \tau$, we have $\bigcup_{u \in \mathcal{U}} u \in \tau$

3) Finite intersections of elements of τ are in τ.

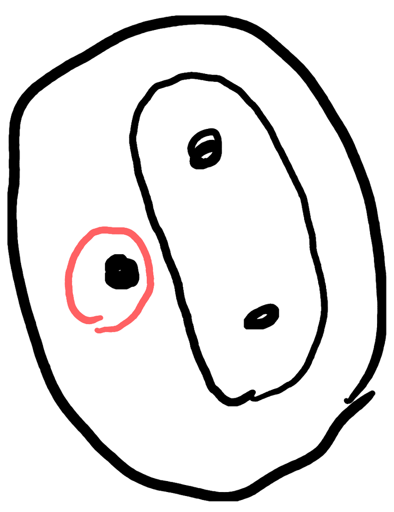
For $\mathcal{U} \subseteq \tau$ with $|\mathcal{U}| < \infty$, we have

$$\bigcap_{u \in \mathcal{U}} u \in \tau$$

Example $X = \{1, 2, 3\}$; $\tau = \{\phi, X, \{1\}, \{2, 3\}\}$

1) ✓ 2) unions of more than one ✓ (with ϕ & X irrelevant)

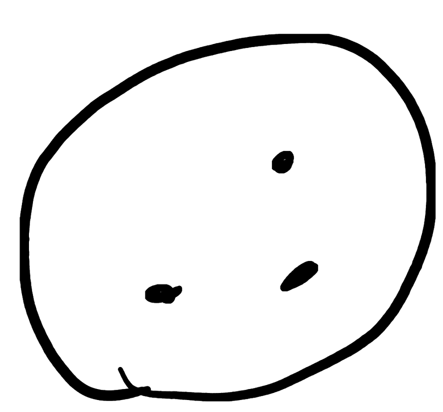
3) sections (all sections are finite) ✓ (sections with ϕ irrelevant)

X 

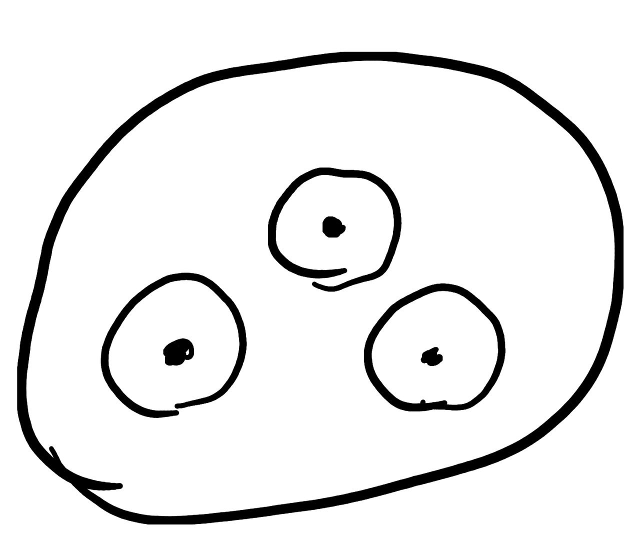
Different topologies possible for a single set X.

trivial topology : X general , $\tau = \{X, \phi\}$
[Klumpentopologie]
[indiscrete topology]

1) ✓ 2) $X \cup \phi = X$ ✓ 3) $X \cap \phi = \phi$ ✓

X 

discrete topology : X general , $\tau = 2^X$ (potent set)
[the set of all subsets]

X 

<u>topological space</u> The pair (X, τ) is called a topological space

<u>open sets</u> An element of τ is called an open set.

Quick check: Does set theoretic definition of <u>open</u> agree with our <u>naive</u> understanding of open intervals?

Take interval $X = [\quad]$ & open subsets

1) is the empty set & X open?

Maybe not naively. Yet, not in conflict with naive understanding. X could not be open in $\mathbb{R}$, which has a different topology. → clarified when introducing metric spaces.

2) unions $[\,(\,(\,)\,)\,] = [\,(\quad)\,]$ ✓ open interval

$[\,(\,) \cup (\,)\,] = [\,(\,)\,(\,)\,]$ ✓ open (no specified boundary)

3) intersections $[\,(\,(\,)\,)\,] = [\,(\,)\,]$ ✓ open interval

$[\,(\,)\,(\,)\,] = [\quad]$ (✓) empty set

closed set

Take an open set U from a top. space. Then the complement of U in X is closed.

[Notation $U^c = X \setminus U$.]

Example: $U = [\;(\;)\;] \Rightarrow U^c = [\;]\;[\;]$ is closed ✓

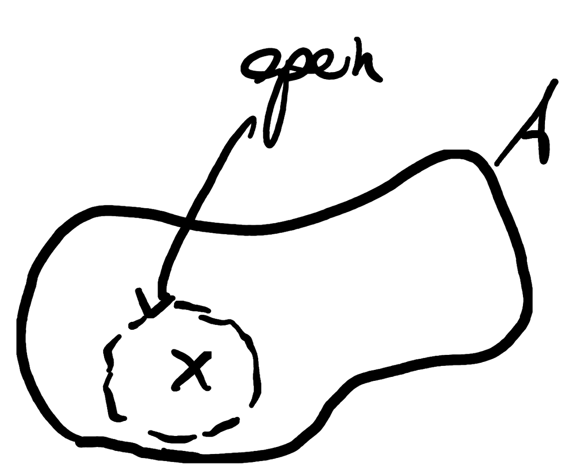
but also $U = X \Rightarrow U^c = \emptyset$ is closed

⇒ There are sets that are both open & closed.

I.2. Additional nomenclature

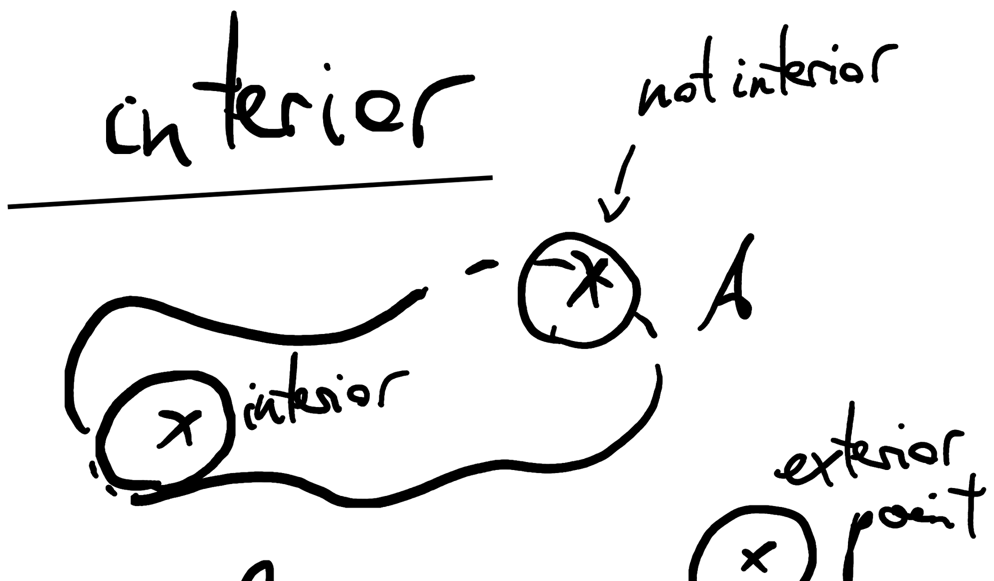
Let (X, τ) be a topological space.

neighorhood



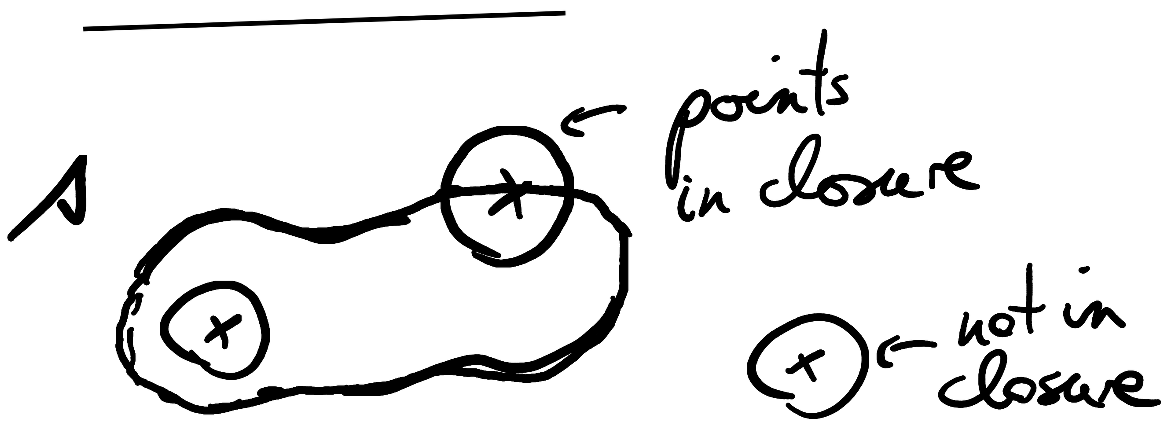
A neighborhood of an $x \in X$ is a subset $A \subseteq X$ that contains an open set with x in it.

interior



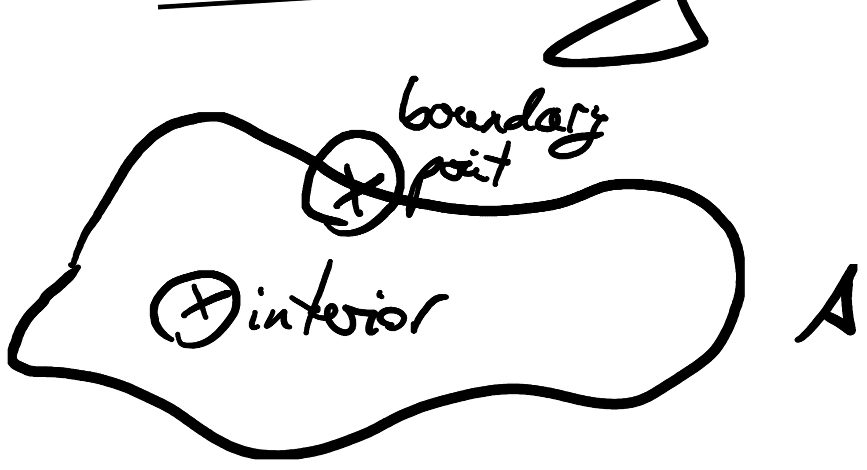
Each $x \in A$ that has a neighborhood fully in A is in the interior of $A \equiv \mathring{A}$.

closure



The closure of A is given by all points that do not have a neighborhood that is disjoint from $A \equiv \bar{A}$.

boundary



The points of A that are in A's closure but not in A's interior form the boundary of A: $\partial A = \bar{A} \setminus A^\circ$.