@ technical staff: Please…

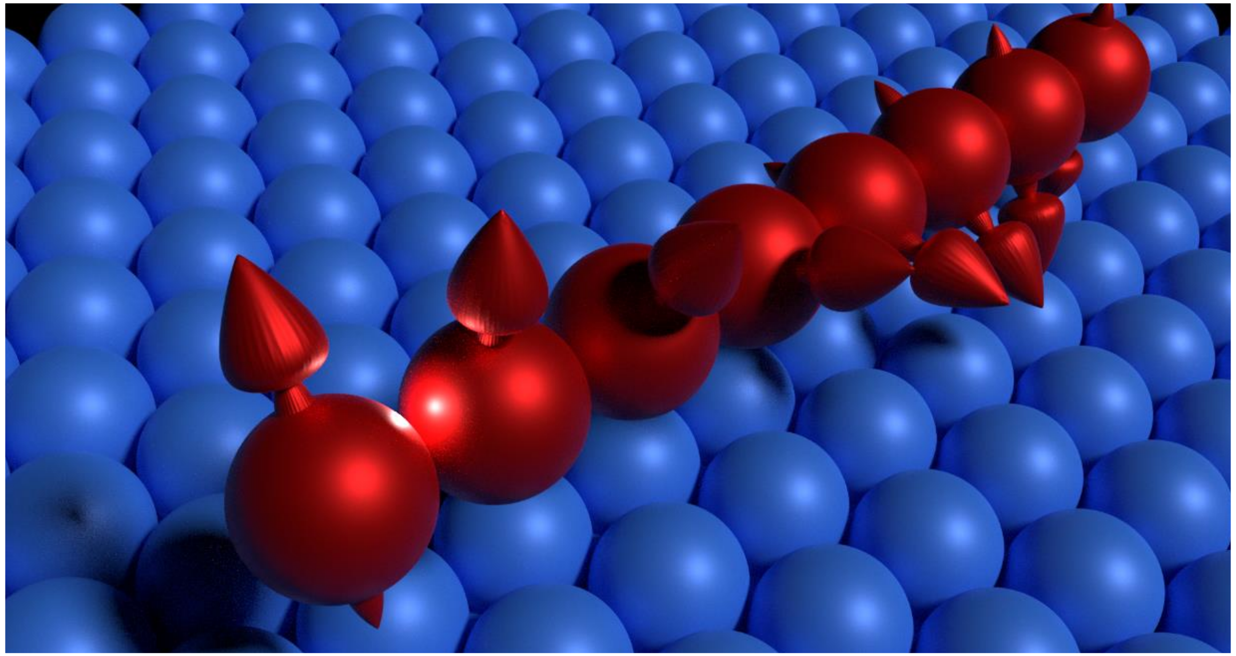
- Turn off microphone or make sure it works
- Unplug everything that can make a beeping noise when connecting and disconnecting
- Laser pointer, stick, or presenting aid available?

Almost classical effects in quantum spin systems

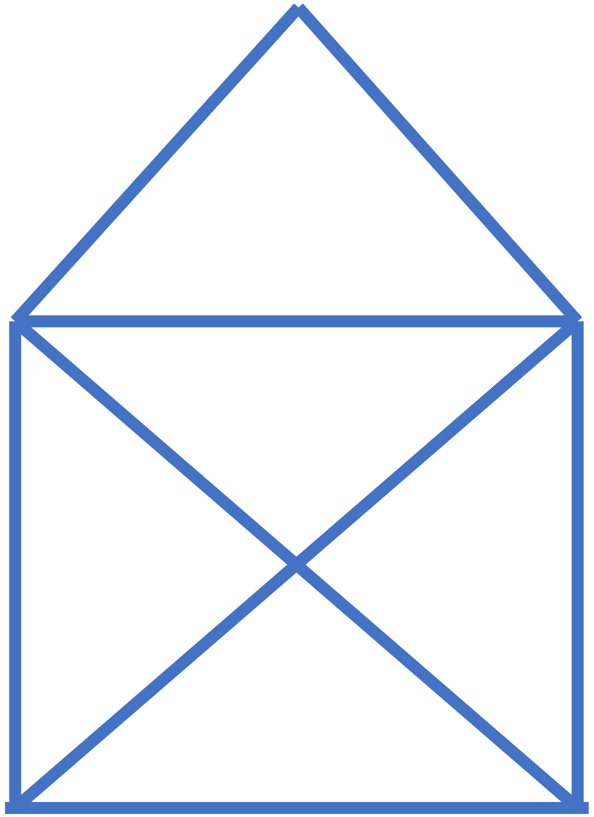
Thore Posske, Universität Hamburg

SPICE Workshop Quantum Functionalities of Nanomagnets

June 17, 2025



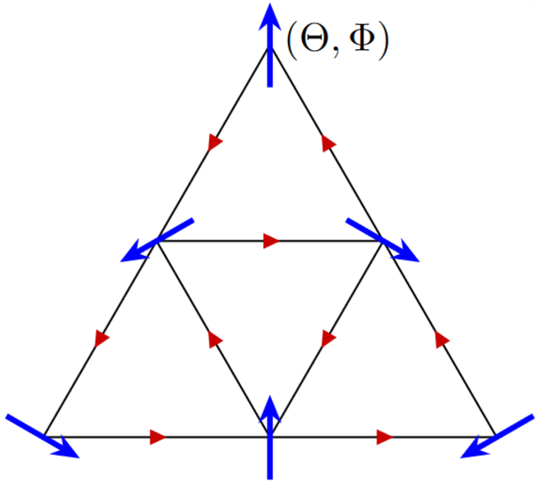
Wake up, brain – Topology

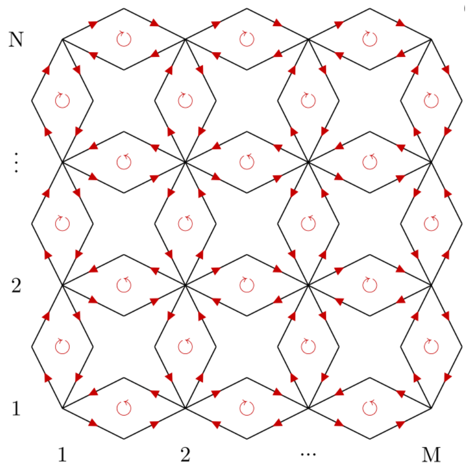


Can you draw the House of Nikolaus?
One continuous stroke, no edge twice, **start from top**.

Outline

- **Product eigenstate method in easy-plane quantum mangets**
- Large degeneracy in Heisenberg models on a general graph

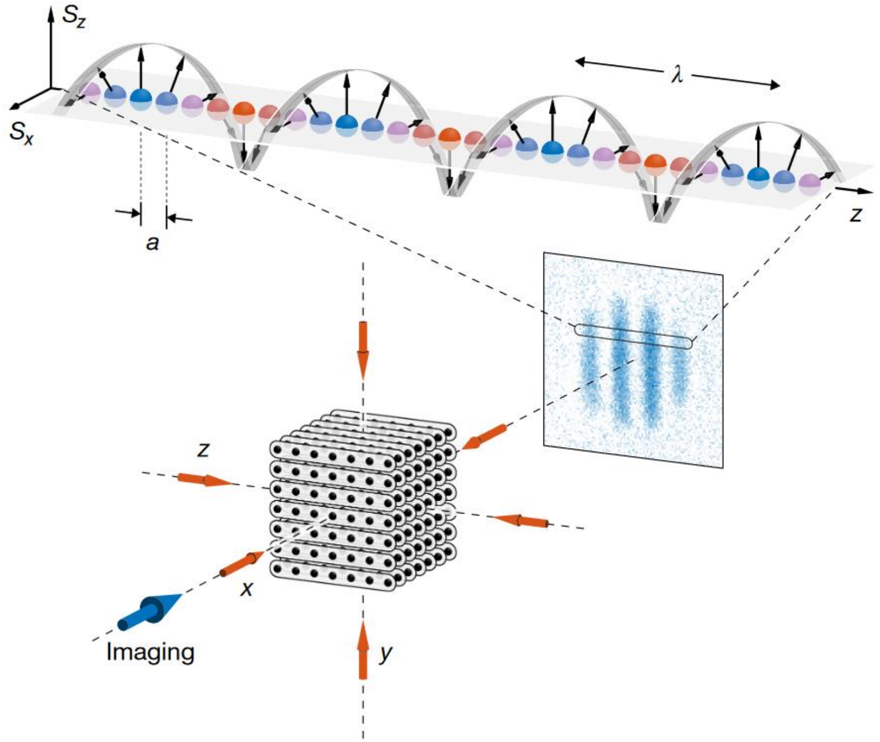




Felix Gerken

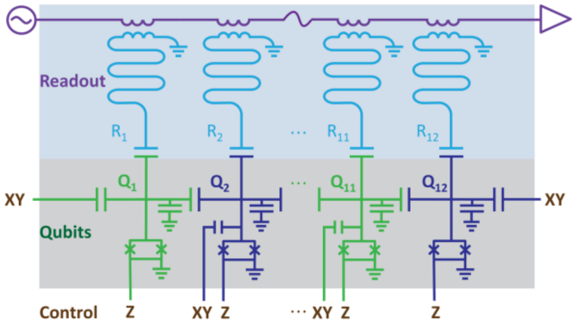
Quantum magnetism - realizations

Cold atoms



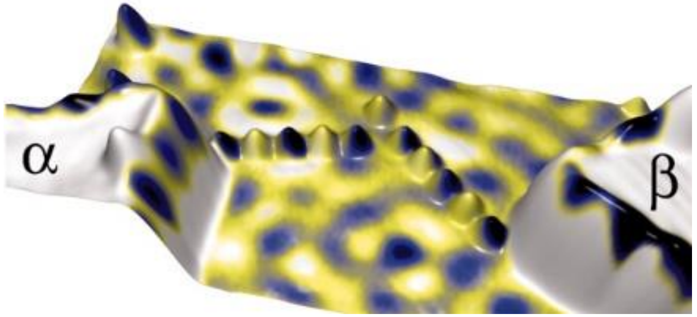
Jepsen [..], Demler & Ketterle
Nature **588**, 403 (2020)

Quantum computers



Gong et al., PRL **112**, 110501 (2019)

Solid states



Khajetoorians, [..], Wiesendanger
Science ***332,*** *1063* (2011)

Hong, [..], Hess
PRL (2024)
Phys. Rev. B **106**, L220406 (2022)

Complete solutions to quantum spin systems

- Integrable systems (Bethe ansatz)
  - Special 1D spin chains [Bethe 1931, Klümper, Göhmann, Frahm, Popkov, ..]
  - Central spin model [Richardson 1963-64, (inhomog.) Bortz, Stoze (2007) (spin S, homog.) Nepomechie 2018]
- Special 2D models
  - Kitaev toric code [Kitaev 2006]
  - Levin-Wen models (string nets) [Levin & Wen 2004]
- Solutions by statistical transmutation
  - Jordan-Wigner transform (spins to fermions) [Jordan & Wigner 1928]
  - Holstein-Primakov transform (spins to bosons) [Holstein & Primakoff 1940]
  - Slave particle methods

Only special systems with complete solution

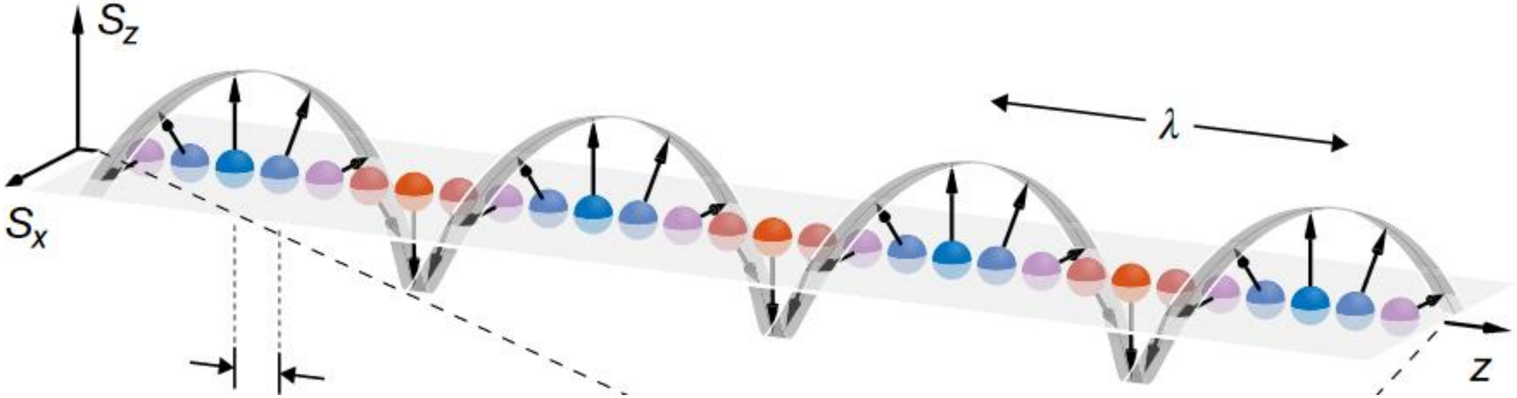
Motivation: Heisenberg model‘s product states

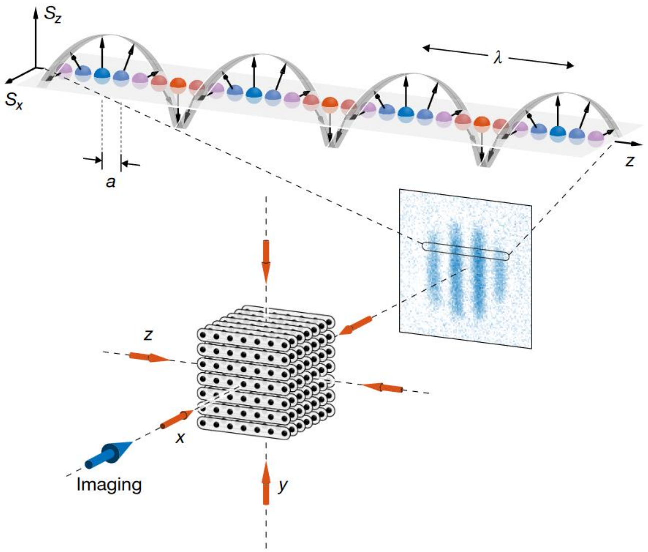
1D Heisenberg model with boundary terms

$$H(\phi) = \sum_{j=2}^{N-2} \left[J(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y) + \Delta S_j^z S_{j+1}^z \right] + J \left[\tilde{\mathbf{S}}_1 \cdot \mathbf{S}_2 + \mathbf{S}_{N-1} \cdot \tilde{\mathbf{S}}_N(\phi) \right].$$

Product eigenstates „Phantom helices

$$|\psi(Q)\rangle = \prod_i \left[\cos(\theta/2) |\uparrow\rangle_i + \sin(\theta/2) \mathrm{e}^{-\mathrm{i}Qz_i} |\downarrow\rangle_i \right]$$





Jepsen [..], Demler & Ketterle
Nature **588**, 403 (2020)

Phantom helices

$$|PH\rangle_1^\pm = \frac{1}{\sqrt{2^N}} \bigotimes_{j=2}^{N-1} (|\uparrow\rangle_j - e^{\pm i(j-2)\gamma} |\downarrow\rangle_j)$$

$$|PH\rangle_2^\pm = \frac{1}{\sqrt{2^N}} \bigotimes_{j=2}^{N-1} (|\uparrow\rangle_j + e^{\pm ij\gamma} |\downarrow\rangle_j)$$

Boundary phase

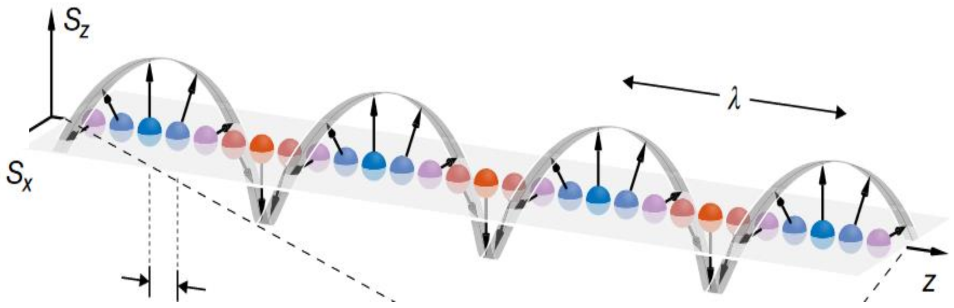
$$\phi_{ph} = \pm(N - M)\pi/3 + \pi\delta_{3,M} \mod 2\pi$$

Motivation

$$|\psi(Q)\rangle = \prod_i \left[\cos(\theta/2)|\uparrow\rangle_i + \sin(\theta/2)\mathrm{e}^{-\mathrm{i}Qz_i}|\downarrow\rangle_i\right]$$

Product eigenstates found in various spin systems

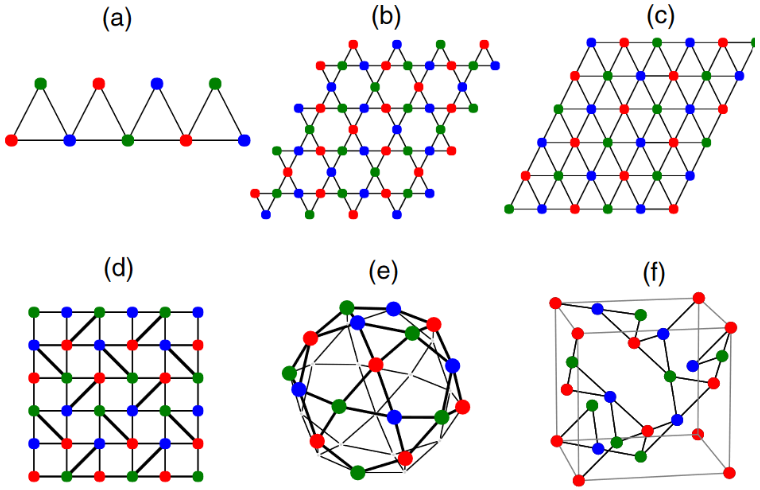
[Cerezo, Rossignoli, … Ríos 2017]
[Pokrovskii, V. L. & Khokhlachev 1975]



1D Phantom helices

[Jepsen .. Ketterle 2022]
[Popkov, Zhang, Göhmann, Klümper … 2020-25]
[Batista & Somma 2012, 2015]

2D Kagome lattice and derivatives



[Changlani .. Fradkin 2018, Fendley 2019]

ND examples

[Jepsen .. Ketterle 2022, Changlani .. Fradkin 2018]

General theory?

All product eigenstates in Heiseberg models from a graphical construction

Felix Gerken[1,2,*], Ingo Runkel[3], Christoph Schweigert[3], and Thore Posske[1,2]

Recently, large degeneracy based on product eigenstates has been found in spin ladders, kagome-like lattices, and motif magnetism, connected to spin liquids, anyonic phases, and quantum scars. We unify these systems by a complete classification of product eigenstates of Heisenberg XXZ Hamiltonians with Dzyaloshinskii-Moriya interaction on general graphs in the form of Kirchhoff rules for spin supercurrents. By this, we construct spin systems with extensive degree of degeneracy linked to exotic condensates which could be studied in atomic gases and quantum spin lattices.


Felix Gerken

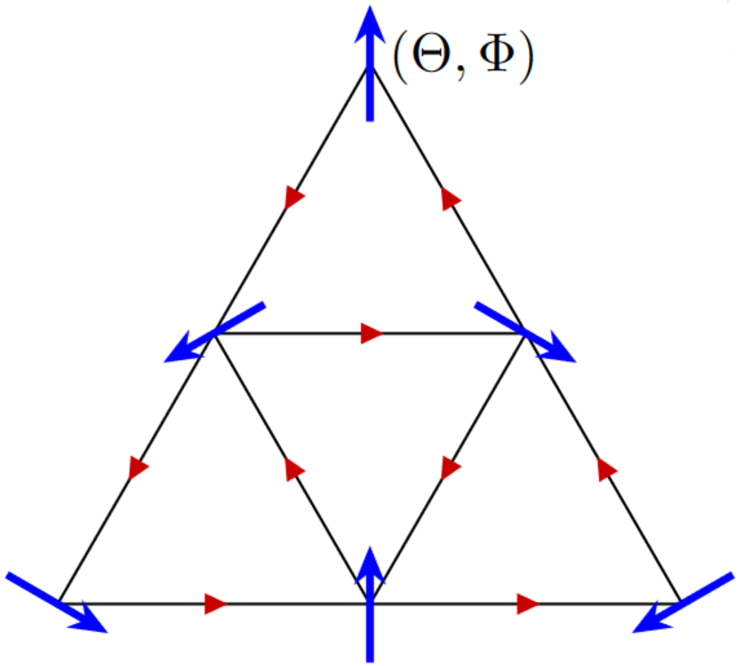

Ingo Runkel

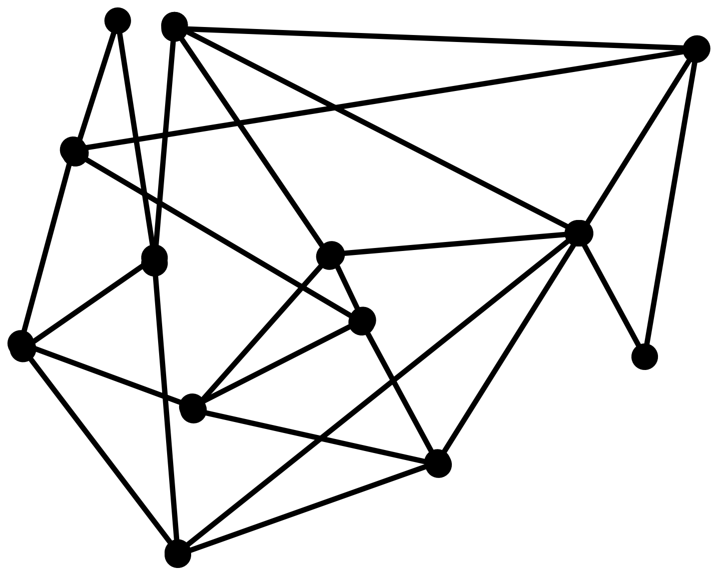

Chr. Schweigert

Phys. Rev. Research **7**, L012008 (2025)

Heisenberg model on a general graph

Graph
vertices k
edges $\{k, l\}$





$$H = \sum_{\{k,l\}} J\, S_k^x S_l^x + J\, S_k^y S_l^y + \Delta\, S_k^z S_l^z + D_{k,l}(S_k^x S_l^y - S_k^y S_l^x)$$

Easy plane magnetism $|J| < |\Delta|$ and Dzyaloshinskii-Moriya interaction D

Symmetries and spin currents

$$H = \sum_{\{k,l\}} J\, S_k^x S_l^x + J\, S_k^y S_l^y + \Delta\, S_k^z S_l^z \qquad [S^a, S^b] = i\,\epsilon_{abc} S^c$$

U(1) spin rotation **symmetry** around z-axis

$$U = e^{i\theta S_{\mathrm{tot}}^{\mathrm{z}}} \text{ with } S_{\mathrm{tot}}^{\mathrm{z}} = \textstyle\sum_j S_j^z$$

Spin-z change

$$\dot{S_j^z} = -i\left[S_j^z, H\right] = i \sum_{\{j,l\}} 2J \left(S_j^x S_l^y - S_j^y S_l^x\right) = i \sum_{\{j,l\}} C_{jl}$$

With the **z-spin current**

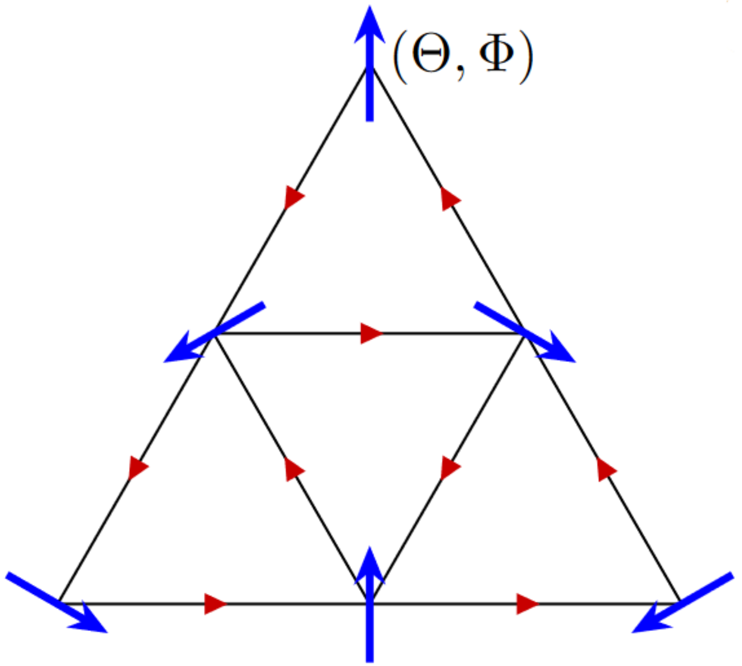
$$C_{jl} = 2J\left(S_j^x S_l^y - S_j^y S_l^x\right)$$

Conservation of spin currents

In eigenstates:

Kirchhoff rule for spin currents

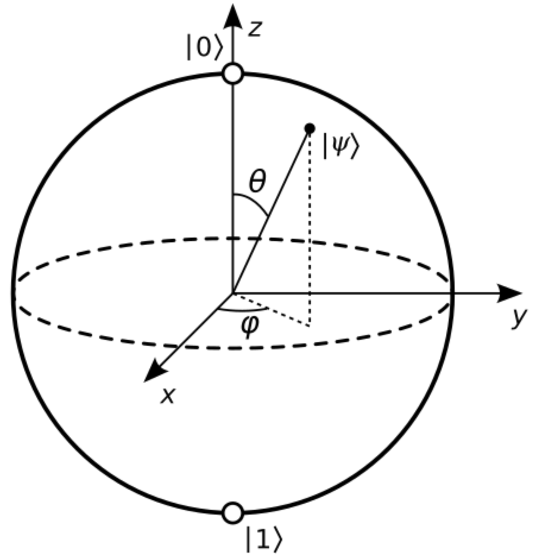
$$\langle \dot{S}_j^z \rangle = \sum_{\{j,l\}} \langle C_{jl} \rangle = 0$$



Product state ansatz

$$H = \sum_{\{k,l\}} J\, S_k^x S_l^x + J\, S_k^y S_l^y + \Delta\, S_k^z S_l^z$$

$$|\Psi(\boldsymbol{\vartheta}, \boldsymbol{\varphi})\rangle = \bigotimes_{i\in V} \left(\cos\left(\frac{\vartheta_i}{2}\right) e^{-\mathrm{i}\frac{\varphi_i}{2}} |\uparrow\rangle_i + \sin\left(\frac{\vartheta_i}{2}\right) e^{\mathrm{i}\frac{\varphi_i}{2}} |\downarrow\rangle_i \right)$$



Coupled trigonometric equations

$$\begin{aligned}
&\frac{4}{\hbar^2} U_j^\dagger U_i^\dagger h_{ij} U_i U_j |\Omega\rangle \\
&= (\cos(\vartheta_i)\cos(\vartheta_j)\Delta + \sin(\vartheta_i)\sin(\vartheta_j)\left(\cos(\varphi_i - \varphi_j) J + \sin(\varphi_i - \varphi_j)\kappa_{ij} D\right)) |\Omega\rangle \\
&+ (\cos(\vartheta_i)\sin(\vartheta_j)\left(\cos(\varphi_i - \varphi_j) J + \sin(\varphi_i - \varphi_j)\kappa_{ij} D\right) - \sin(\vartheta_i)\cos(\vartheta_j)\Delta - \mathrm{i}\sin(\vartheta_j)\left(\sin(\varphi_i - \varphi_j) J - \cos(\varphi_i - \varphi_j)\kappa_{ij} D\right)) |i\rangle \\
&+ (\cos(\vartheta_j)\sin(\vartheta_i)\left(\cos(\varphi_i - \varphi_j) J + \sin(\varphi_i - \varphi_j)\kappa_{ij} D\right) - \sin(\vartheta_j)\cos(\vartheta_i)\Delta + \mathrm{i}\sin(\vartheta_i)\left(\sin(\varphi_i - \varphi_j) J - \cos(\varphi_i - \varphi_j)\kappa_{ij} D\right)) |j\rangle \\
&+ (\sin(\vartheta_i)\sin(\vartheta_j)\Delta + (\cos(\vartheta_i)\cos(\vartheta_j) - 1)\left(\cos(\varphi_i - \varphi_j) J + \sin(\varphi_i - \varphi_j)\kappa_{ij} D\right)) |i, j\rangle \\
&+ \mathrm{i}\left((\cos(\vartheta_i) - \cos(\vartheta_j))\left(\sin(\varphi_i - \varphi_j) J - \cos(\varphi_i - \varphi_j)\kappa_{ij} D\right)\right) |i, j\rangle.
\end{aligned}$$

$$\begin{aligned}
\frac{2}{\hbar} U_i^\dagger h_i U_i |\Omega\rangle &= \left(\sin(\vartheta_i)\cos(\varphi_i) B_i^x + \sin(\vartheta_i)\sin(\varphi_i) B_i^y + \cos(\vartheta_i) B_i^z\right) |\Omega\rangle \\
&+ \left(\cos(\vartheta_i)\cos(\varphi_i) B_i^x + \cos(\vartheta_i)\sin(\varphi_i) B_i^y - \sin(\vartheta_i) B_i^z - \mathrm{i}\left(\sin(\varphi_i) B_x - \cos(\varphi_i) B_y\right)\right) |i\rangle.
\end{aligned}$$

$$\begin{aligned}
\varepsilon(\boldsymbol{\vartheta}, \boldsymbol{\varphi}) = &\frac{\hbar^2}{4} \sum_{\{i,j\}\in E} \left(\cos(\vartheta_i)\cos(\vartheta_j)\Delta + \sin(\vartheta_i)\sin(\vartheta_j)\left(\cos(\varphi_i - \varphi_j) J + \sin(\varphi_i - \varphi_j)\kappa_{ij} D\right)\right) \\
&+ \frac{\hbar}{2} \sum_{i\in V} \left(\sin(\vartheta_i)\cos(\varphi_i) B_i^x + \sin(\vartheta_i)\sin(\varphi_i) B_i^y + \cos(\vartheta_i) B_i^z\right).
\end{aligned}$$

Product state ansatz – results

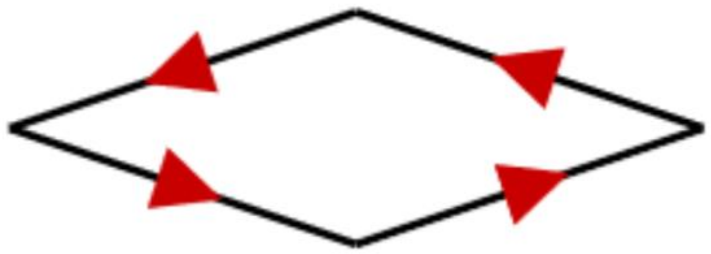
$$|\Psi(\boldsymbol{\vartheta},\boldsymbol{\varphi})\rangle = \bigotimes_{i\in V}\left(\cos\left(\frac{\vartheta_i}{2}\right)e^{-\mathrm{i}\frac{\varphi_i}{2}}|\uparrow\rangle_i + \sin\left(\frac{\vartheta_i}{2}\right)e^{\mathrm{i}\frac{\varphi_i}{2}}|\downarrow\rangle_i\right)$$

Findings:

- Polar angle constant throughout system $\vartheta_i = \Theta$
- **Adjacent azimuthal angles change by $\pm\gamma$** with $\gamma = \arccos\left(\frac{\Delta}{J}\right)$
  - Spin current constant magnitude on every edge
- All product eigenstates degenerate $\epsilon = \frac{\hbar^2\Delta}{4} \times \#edges$

Circuit rule

$$\sum_{loop} \varphi_{j+1} - \varphi_j = 2\,\pi\,n$$

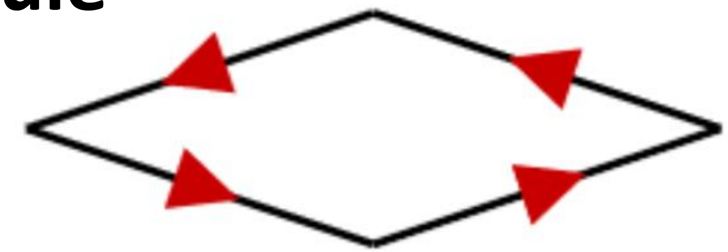


Analogue to quantized flux through plaquettes

Product eigenstates from conserved spin currents

Kirchhoff rule

Circuit rule

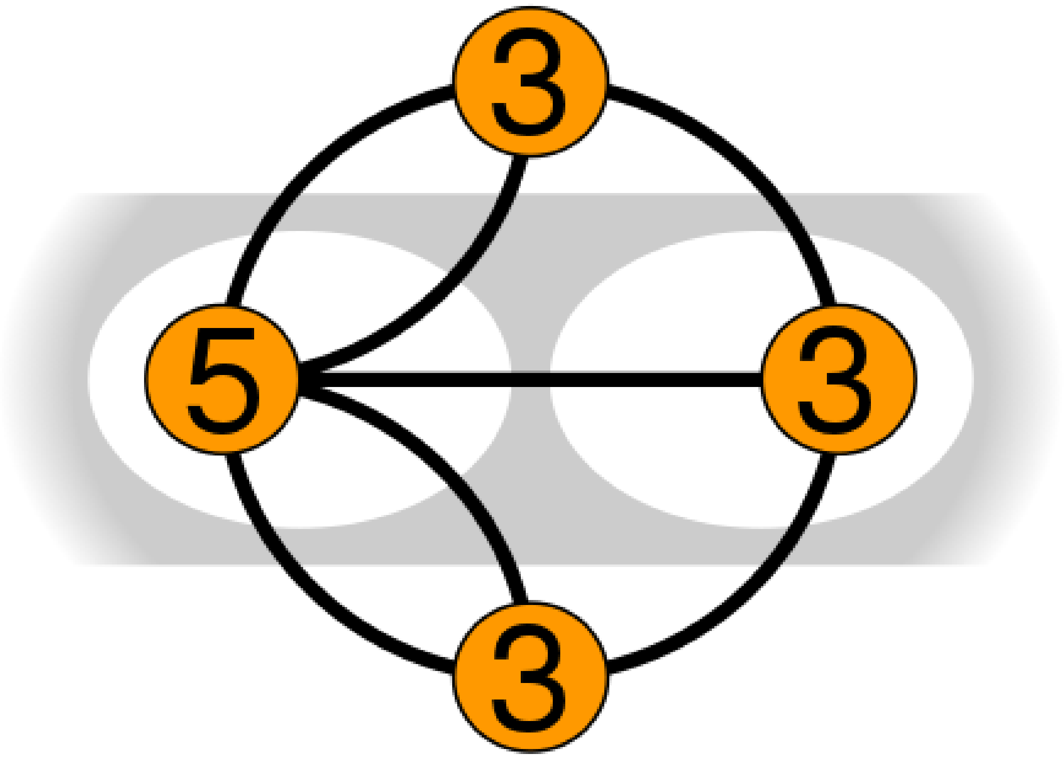


Consistent spin current pattern ⇔ product eigenstate

Complete product eigenstate classification
of XXZ Heisenberg models on graphs

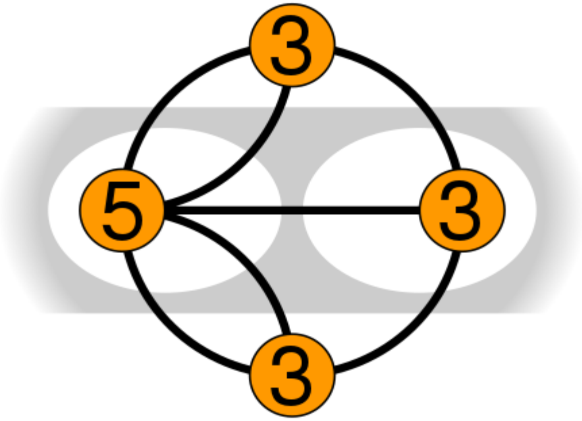
Euler cycles

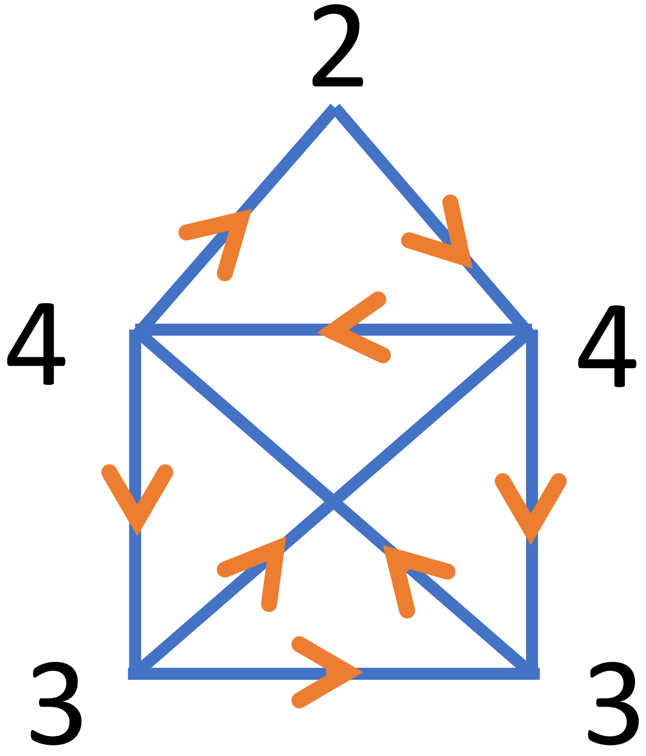
Königsberg bridge problem



Hierholzer, Carl (1873)
Mathematische Annalen, **6** (1): 30–32 (1873)

Euler cycles





Impossible to draw House of Nikolaus starting from the top

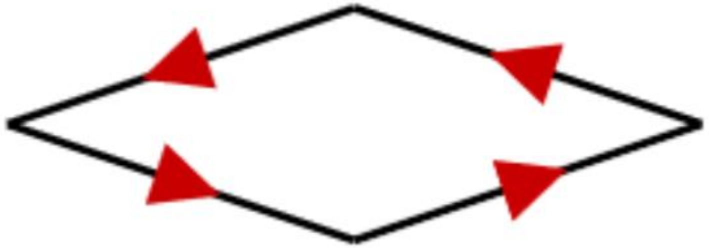
Euler cycles

Kirchhoff rule +
even vertices

⇔

Path through graph exists that travels each edge exactly once.

Circuit rule



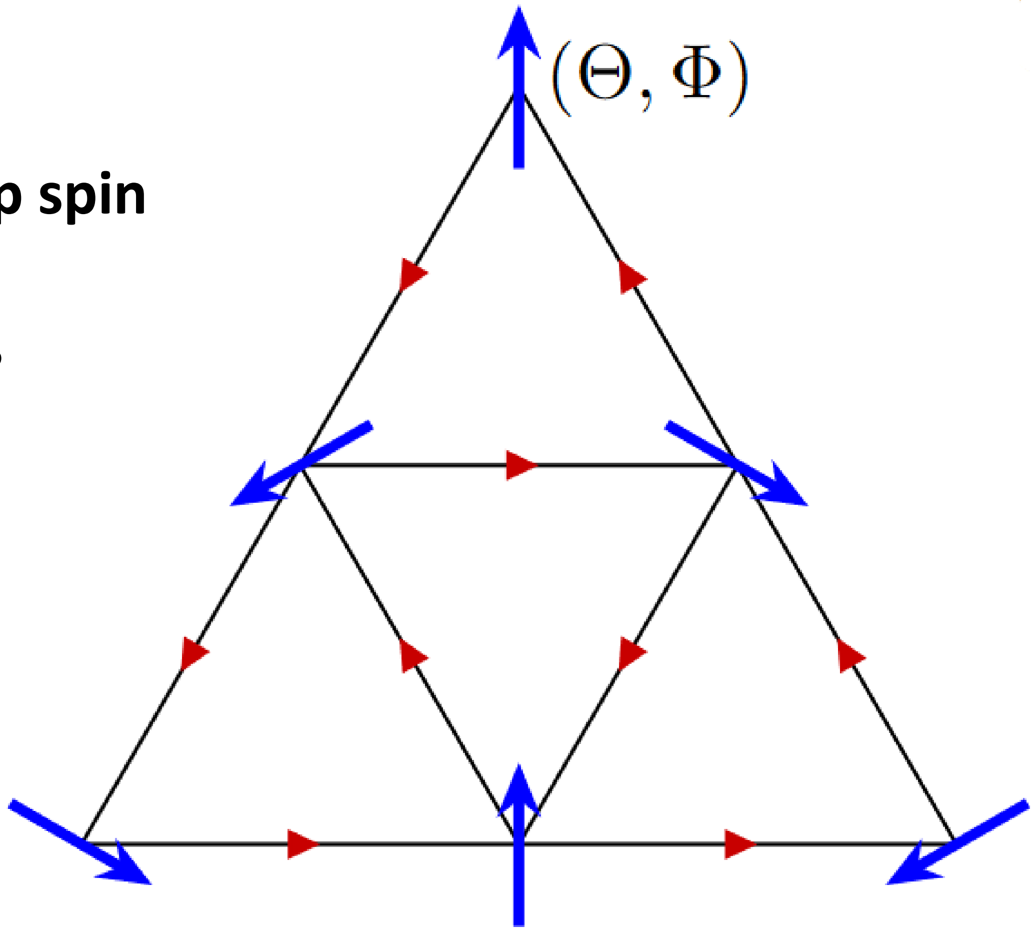
⇔

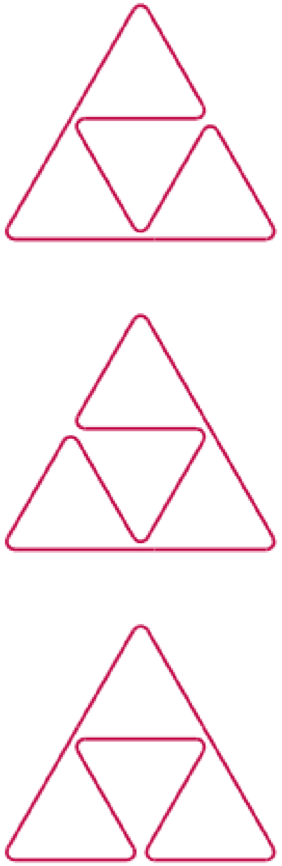
Choose angle
γ appropriate to lattice

Example product eigenstate and Euler cycles

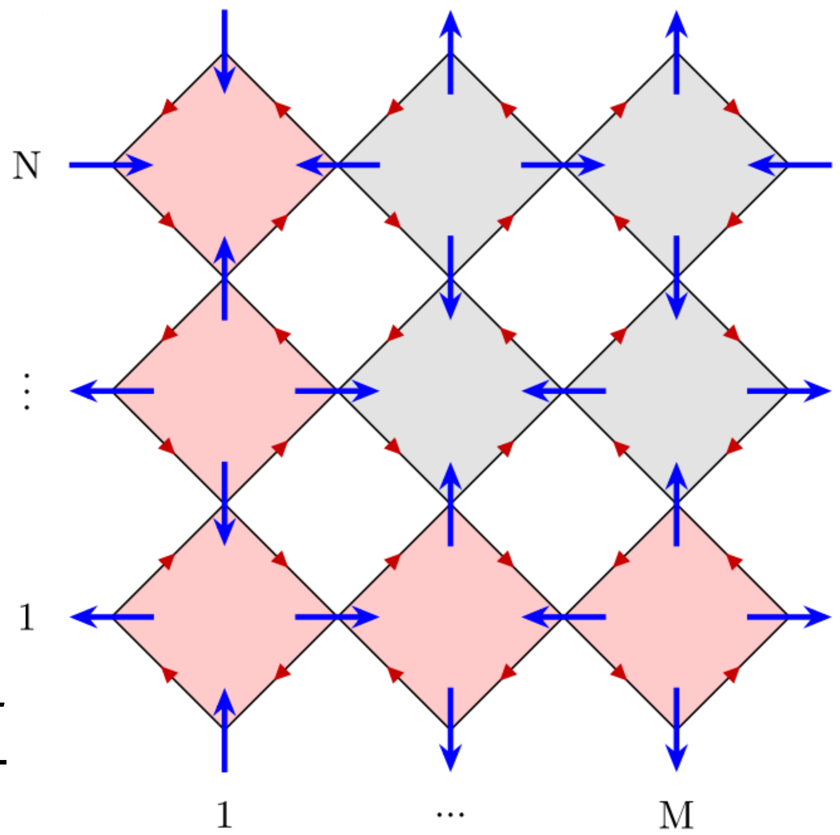
Wrapped up spin spiral along Euler cycles

$$\gamma = \frac{2\pi}{3}$$



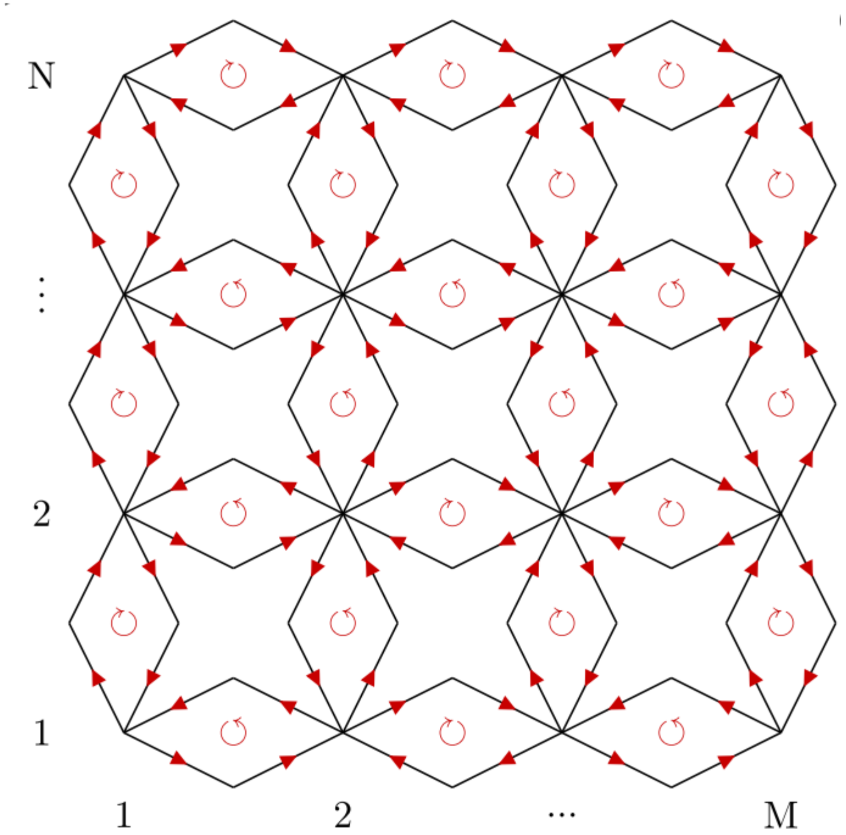


Large degenerace product eigenspaces on special graphs



$\gamma = \frac{\pi}{2}$

Degeneracy $> 2^{M+N+1}$



$\gamma = \frac{\pi}{2}$

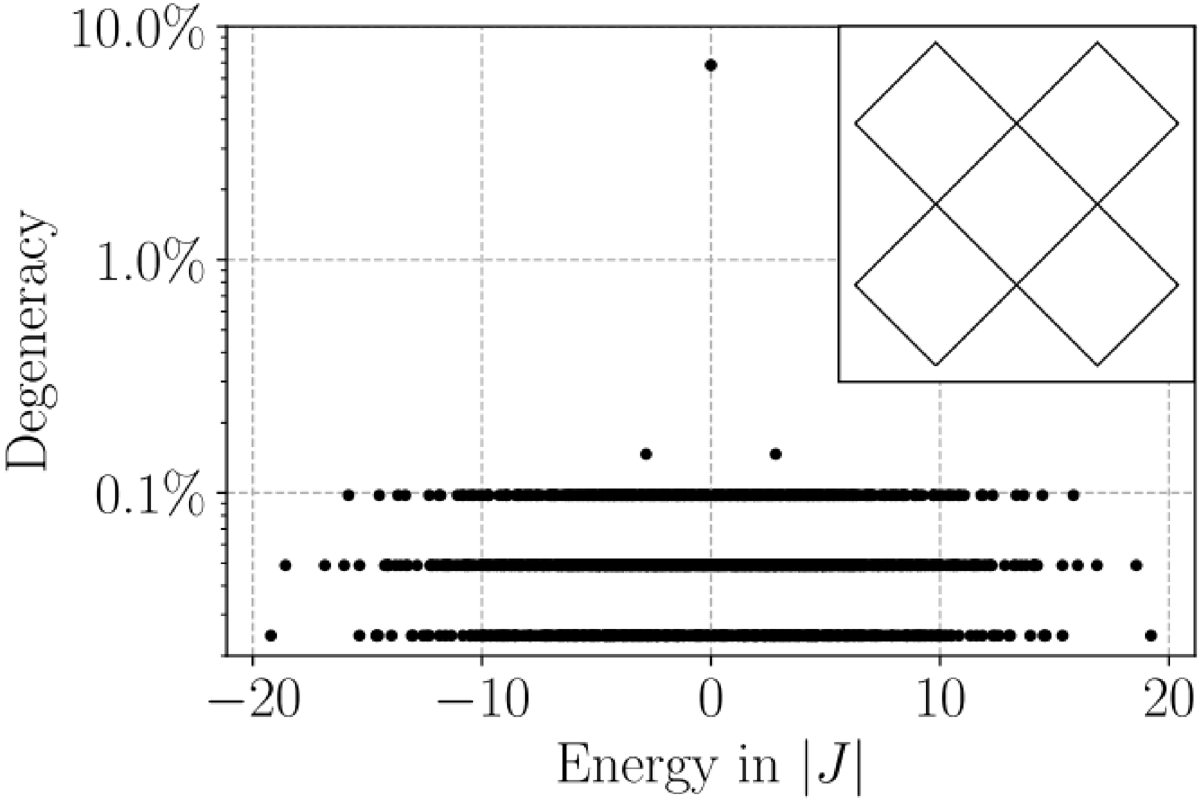
Degeneracy $> 2^{2NM+N+M}$

Mid-spectrum condensate

Macroscopic mid-spectrum degeneracy at energy

$$\epsilon = \frac{\hbar^2 \Delta}{4} \times \#edges$$

Not all degeneracy explained by product states



Hidden anyonic condensate?

PRL **109,** 227203 (2012) PHYSICAL REVIEW LETTERS week ending 30 NOVEMBER 2012

Condensation of Anyons in Frustrated Quantum Magnets

C. D. Batista* and Rolando D. Somma

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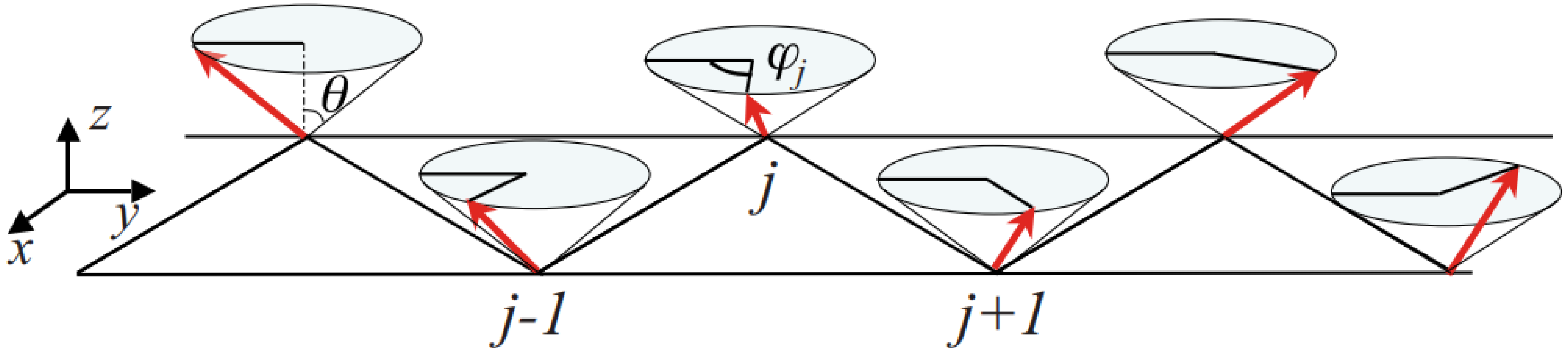
(Received 23 June 2012; published 28 November 2012)



Maps spins to anyons

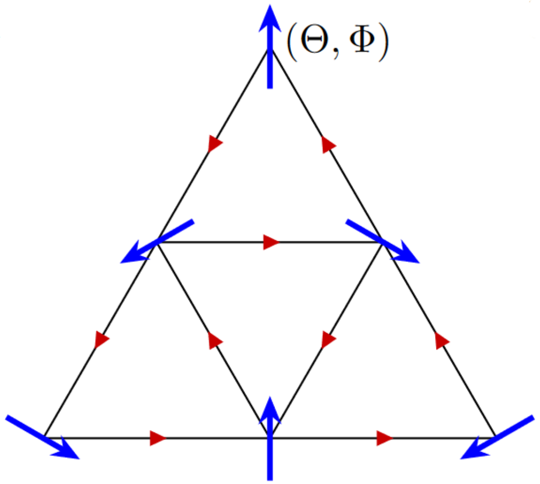
$$a_j^\dagger = \exp\{i\phi \textstyle\sum_{l<j}(s_l^z + 1/2)\} s_j^+$$

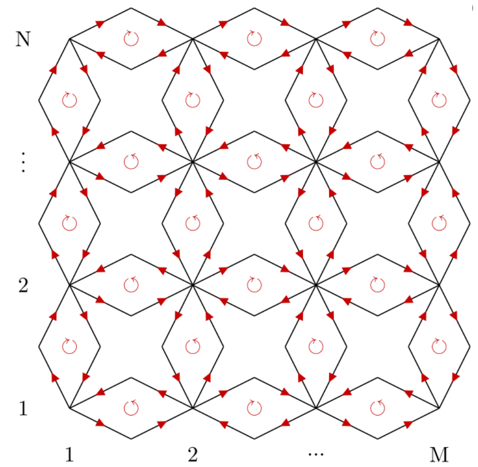
Anyon condensate

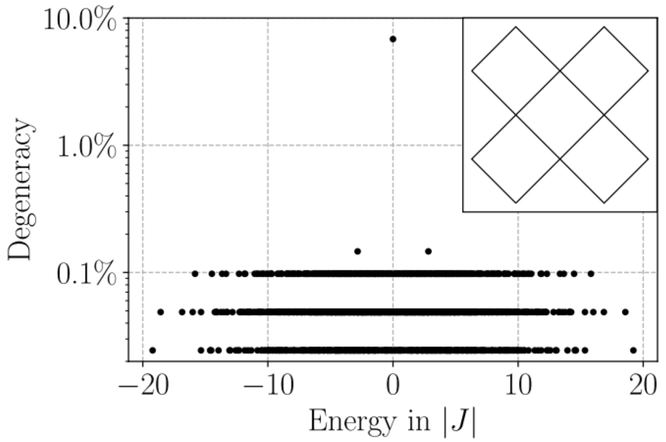
$$|\psi_{n,m}(Q)\rangle = \lim_{\phi \to -4Q} c_\phi (\bar{a}_Q^\dagger)^n (\bar{a}_{-Q}^\dagger)^m |\varnothing\rangle$$

Conclusions and outlook

- **Flexible general theory of product eigenstate in easy-plane quantum mangets**
- **Degenerate spin states at ground state and finite energy**
- Preparation schemes and physical properties of mid-spectrum condensates
- Connection to hardcore anyons **beyond product states**
- New vacua for Bethe ansatz and semi-analytical ansätze







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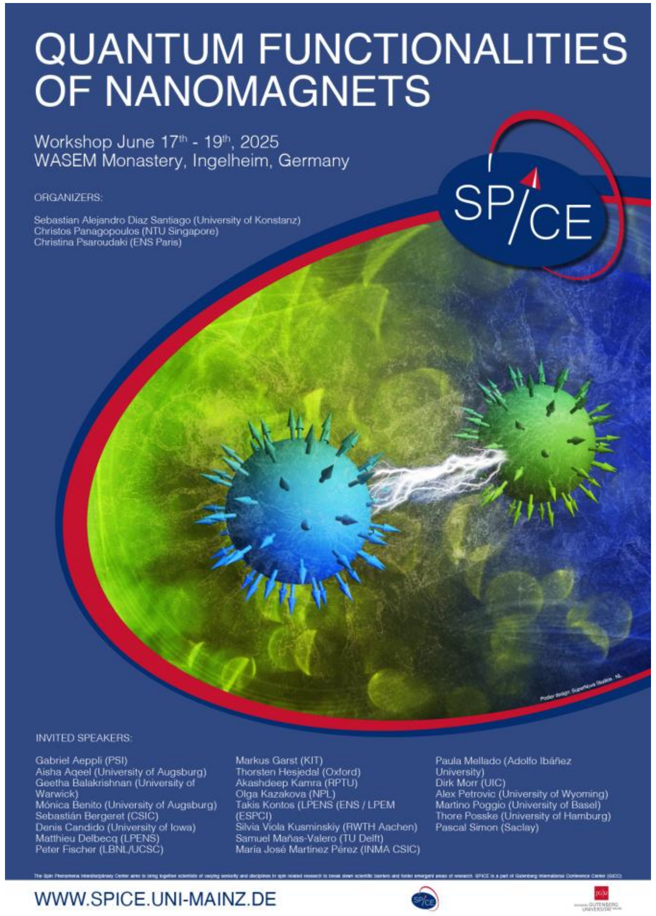
Thank you for the invitation

Danke



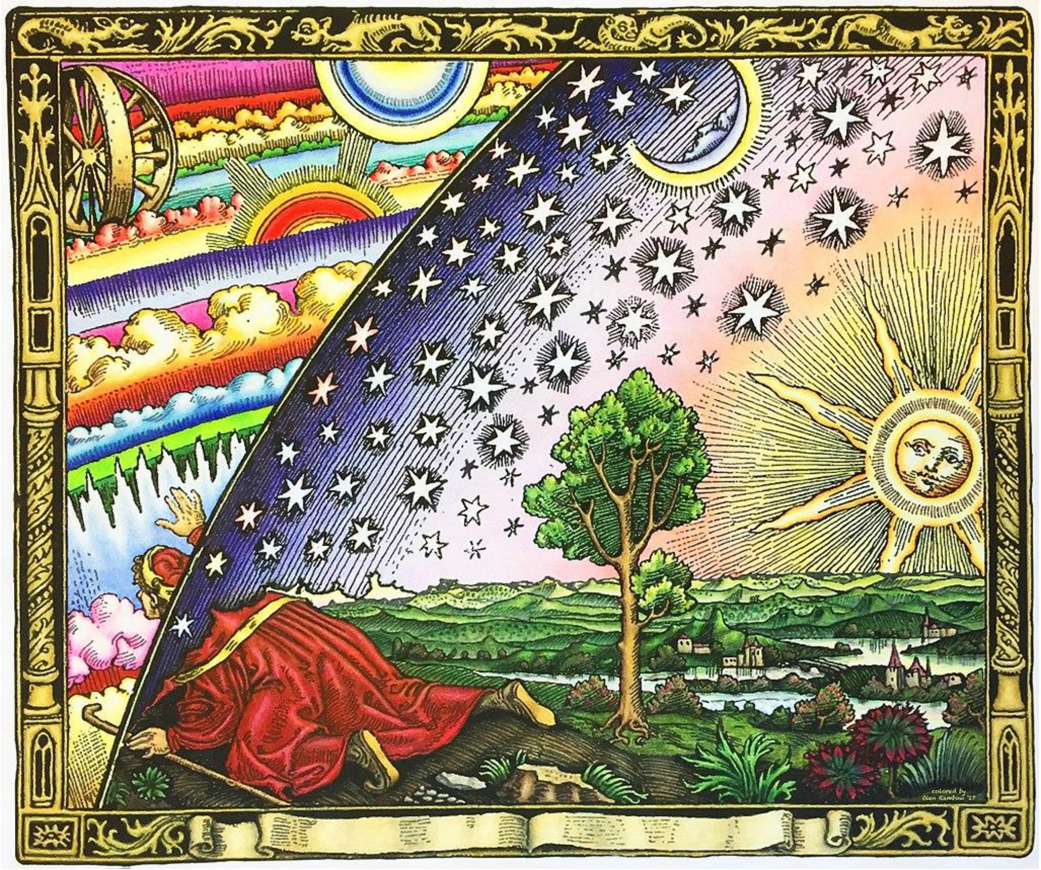
Thank you

Funding

erc Starting grant
QUANTWIST

DFG Deutsche
Forschungsgemeinschaft

UHH EXZELLENZCLUSTER
CUI: ADVANCED
IMAGING OF MATTER

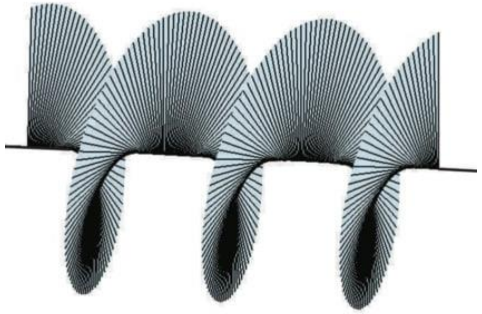


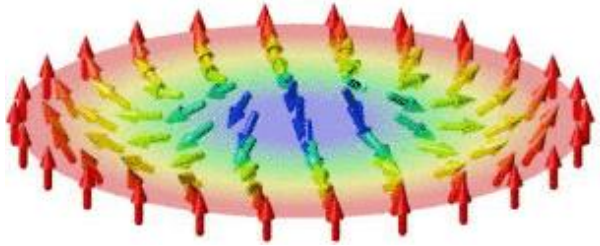
Flammarion engraving, Paris 1888
„Die Neugier hinter die Dinge zu schauen“

Remarks about boundary control

Topological classification, beyond 2D
Homotopy groups of spheres

	S^1	S^2	S^3	S^4	S^5	S^6	S^7	S^8
S^0	0	0	0	0	0	0	0	0
S^1	$\mathbb{Z}$	0	0	0	0	0	0	0
S^2	0	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_{12}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$





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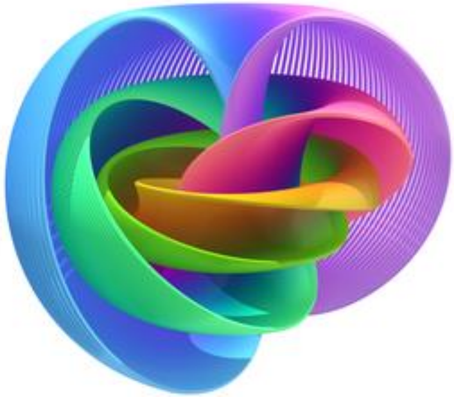
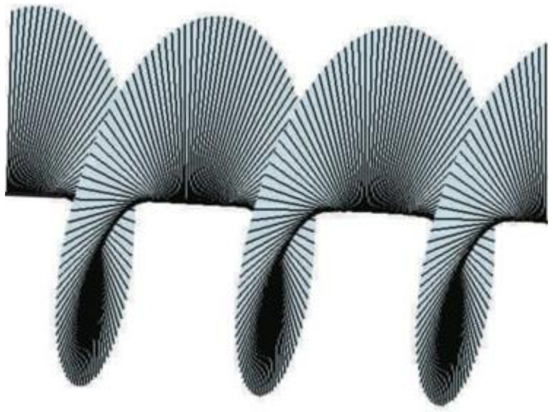
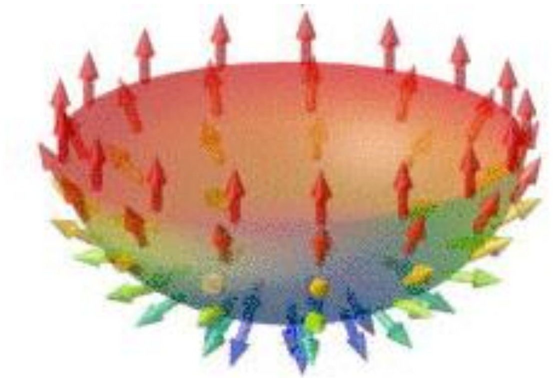


Topology and boundary effects



$L = (0,1)$

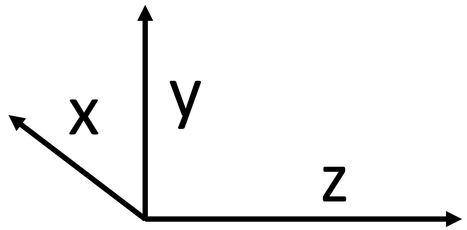
	L^1	L^2	L^3	L^4	L^5	L^6	L^7	L^8
S^0	0	0	0	0	0	0	0	0
S^1	0	0	0	0	0	0	0	0
S^2	0	0	0	0	0	0	0	0



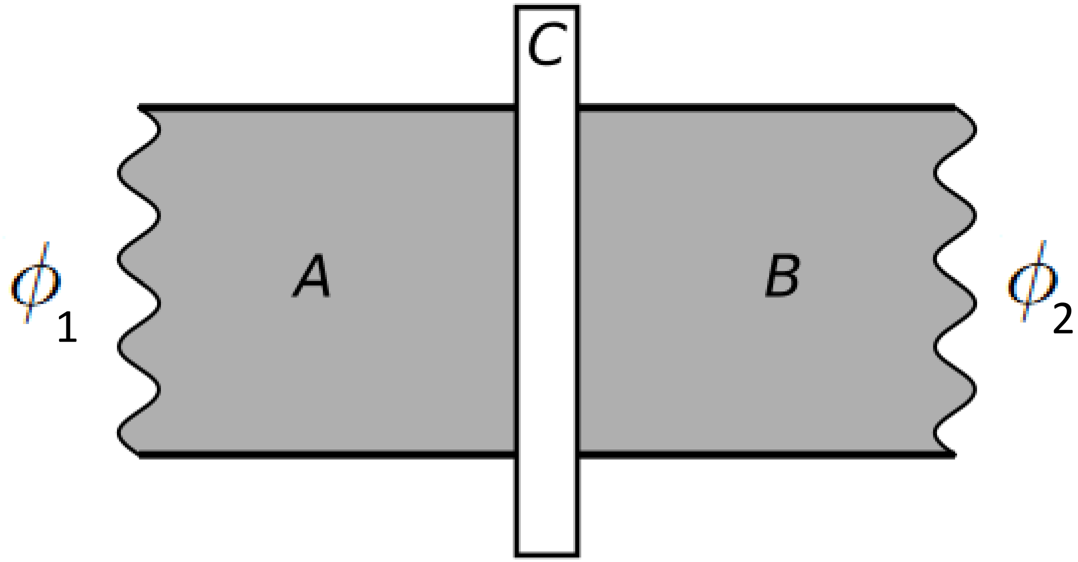
Boundaries essential for topological protection

Generalized Josephson effect

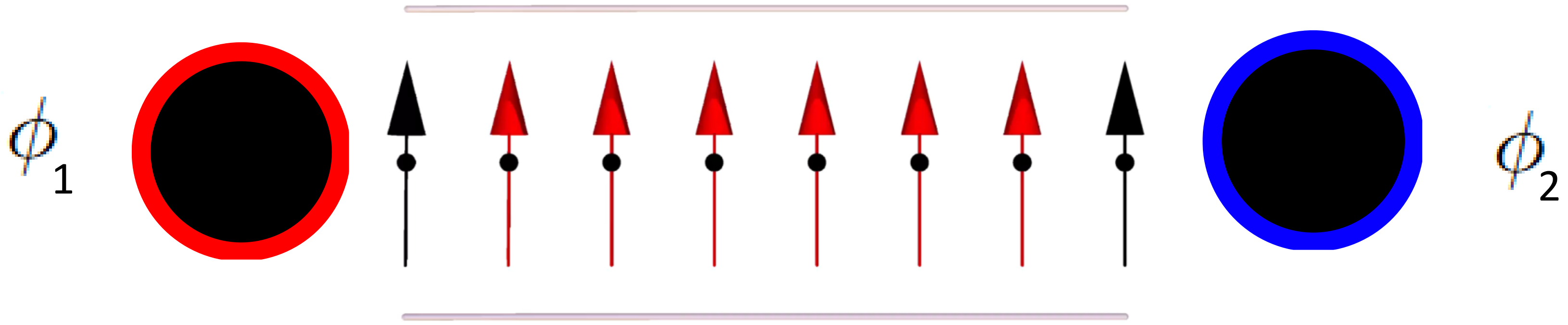
U_1 symmetry broken at boundaries



Original



Generalized



Generalized Josephson effect with arbitrary periodicity in quantum magnets

Anshuman Tripathi,[1] Felix Gerken,[1,2] Peter Schmitteckert,[3] Michael Thorwart,[1,2] Mircea Trif,[4] and Thore Posske[1,2]

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[2] *The Hamburg Centre for Ultrafast Imaging, Hamburg, Germany*

[3] *HQS Quantum Simulations GmbH, Karlsruhe, Germany*

[4] *International Research Centre MagTop, Institute of Physics, Polish Academy of Sciences, Warsaw, Poland*

Easy-plane quantum magnets are strikingly similar to superconductors, allowing for spin supercurrent and an effective superconducting phase stemming from their $U(1)$ rotation symmetry around the z-axis. We uncover a generalized fractional Josephson effect with a periodicity that increases linearly with system size in one-dimensional spin-1/2 chains at selected anisotropies and phase-fixing boundary fields. The effect combines arbitrary integer periodicities in a single system, exceeding the 4π and 8π periodicity of superconducting Josephson effects of Majorana zero modes and other exotic quasiparticles. We reveal a universal energy-phase relation and connect the effect to the recently discovered phantom helices.

Anshuman Tripathi

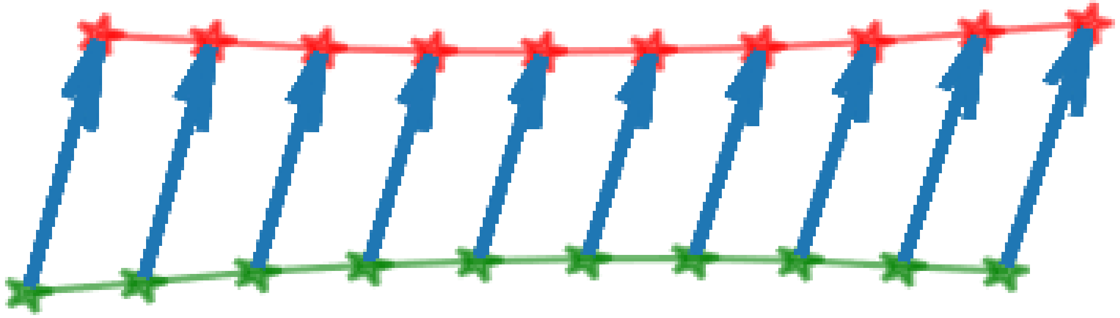
Felix Gerken

Peter Schmitteckert

Michael Thorwart

Mircea Trif

Generalized high-periodicity Josephson effect

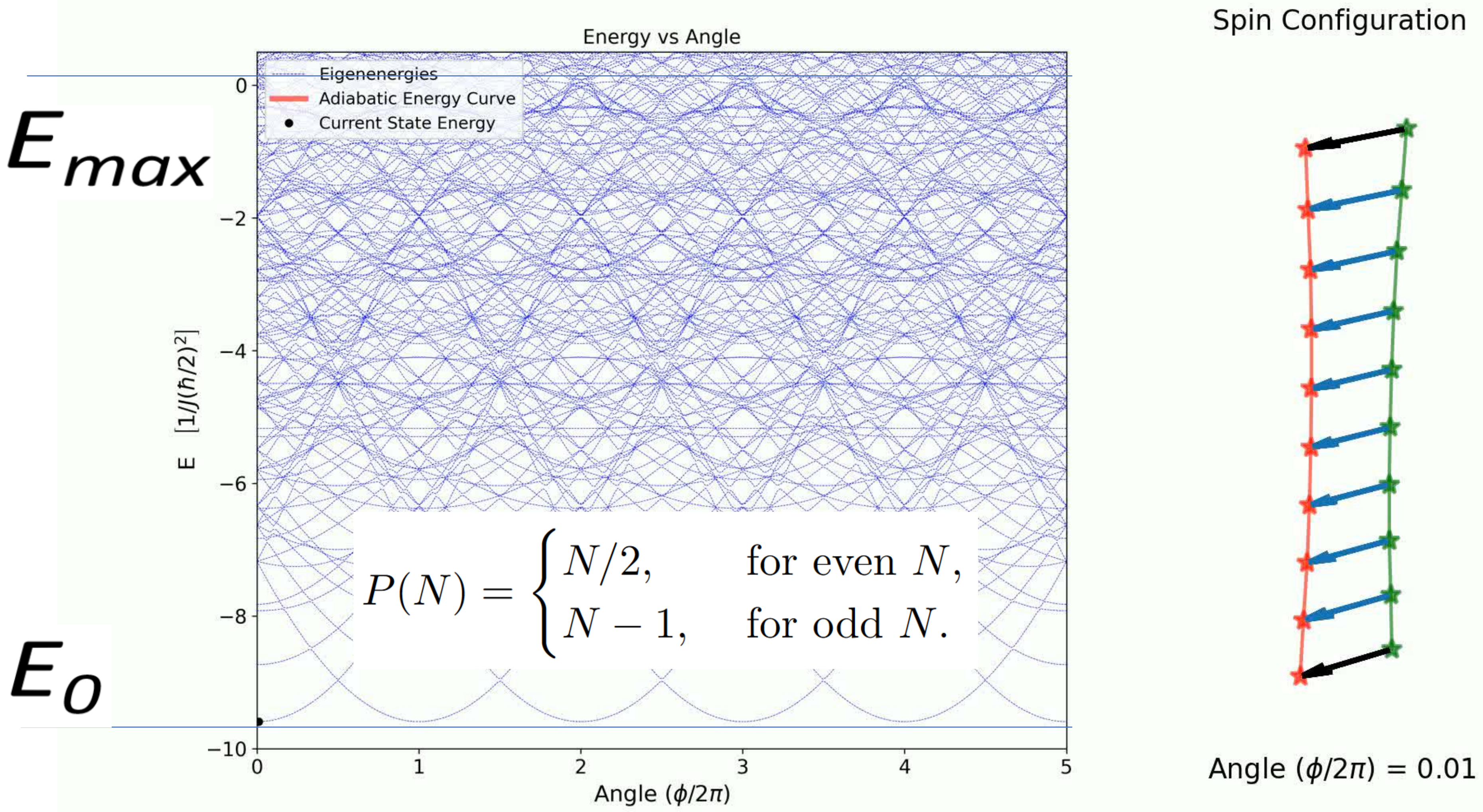


$$H(\phi) = \sum_{j=2}^{N-2} \left[J(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y) + \Delta S_j^z S_{j+1}^z \right]$$

$$+ J \left[\tilde{\mathbf{S}}_1 \cdot \mathbf{S}_2 + \mathbf{S}_{N-1} \cdot \tilde{\mathbf{S}}_N(\phi) \right].$$

Delta = 1/2

Generalized high-periodicity Josephson effect

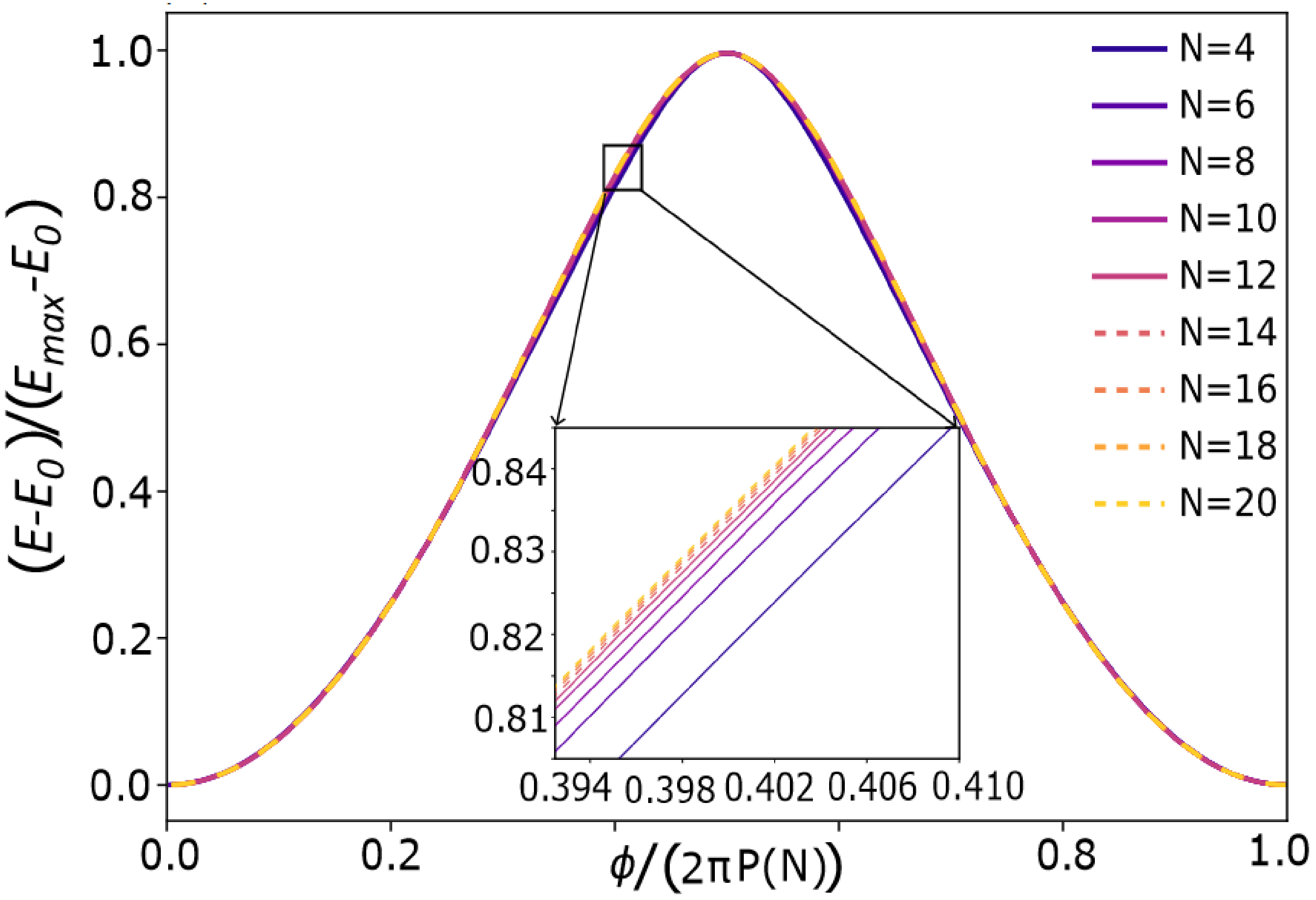


$$P(N) = \begin{cases} N/2, & \text{for even } N, \\ N-1, & \text{for odd } N. \end{cases}$$

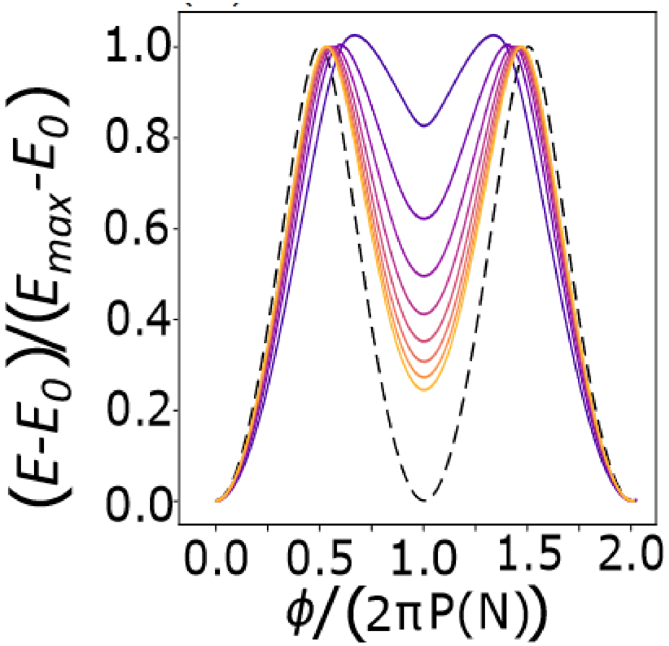
$$H(\phi) = \sum_{j=2}^{N-2} \left[J(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y) + \Delta S_j^z S_{j+1}^z \right] + J \left[\tilde{\mathbf{S}}_1 \cdot \mathbf{S}_2 + \mathbf{S}_{N-1} \cdot \tilde{\mathbf{S}}_N(\phi) \right].$$

Universal phase-energy relation

Even chain lengths

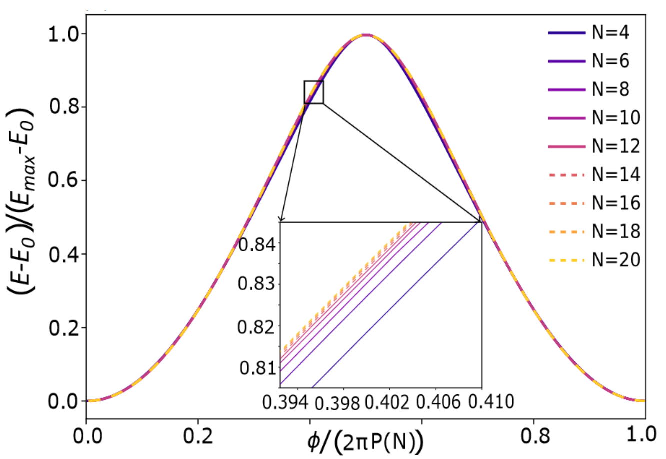


Odd chain lengths



Universal model in Fourier space

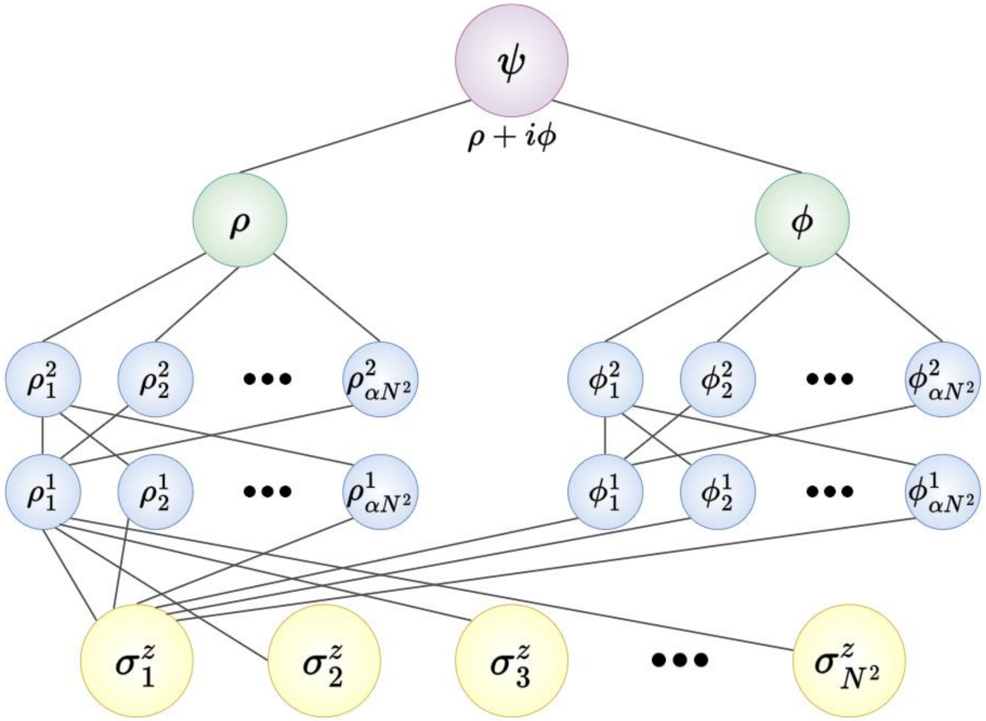
$$\frac{H - E_0}{E_{\max} - E_0} = \sum_{h \in Q} \alpha_h |h\rangle\langle h| + \sum_{\varepsilon \neq h} \alpha_\varepsilon |\varepsilon\rangle\langle \varepsilon|$$

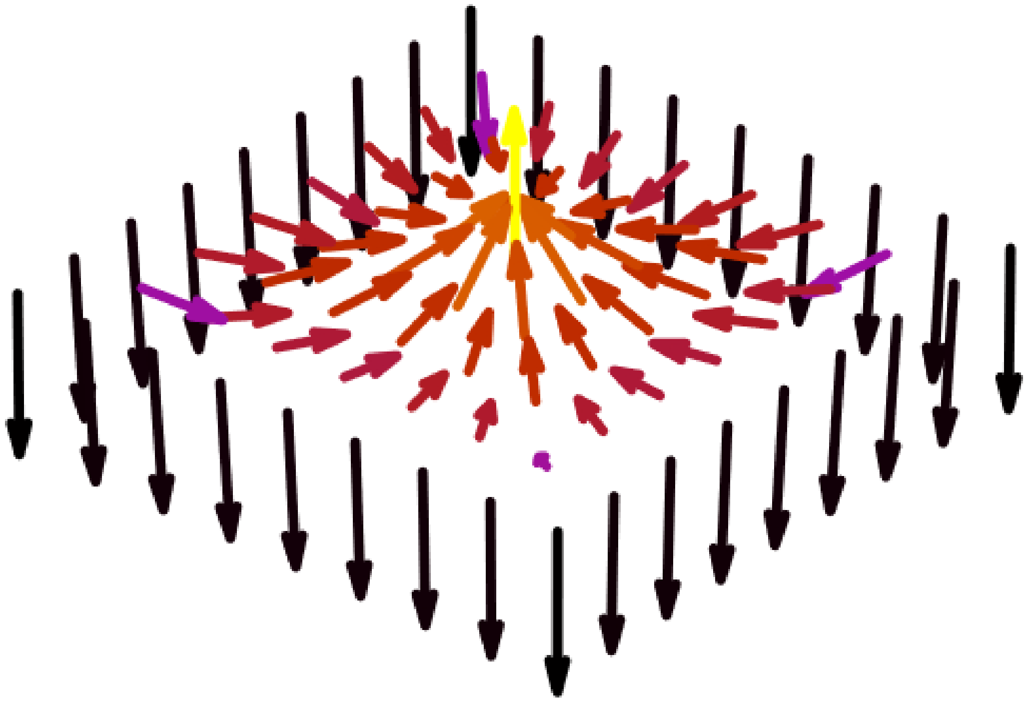


$$\frac{H - E_0}{E_{\max} - E_0} = \sum_{k,k' \in K} \gamma(k' - k)|k'\rangle\langle k| + R$$

$$|k\rangle = \sum_{h \in Q} e^{-ikh}|h\rangle/\sqrt{P}$$

Artificial neural network quantum states



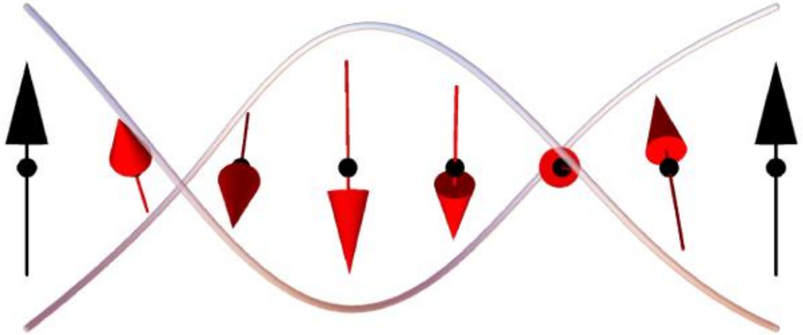


Joshi, Peters, Posske, PRB (2023), PRR (2023), Arxiv (2024)

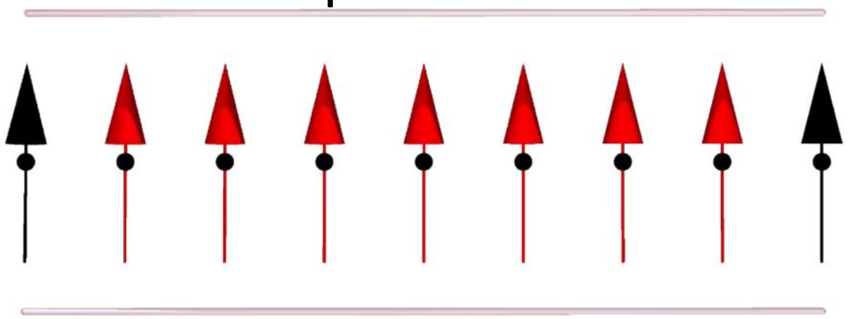
Quantum spin helices

$$\mathcal{H}(t) = \sum_{i=2}^{n-2} J\left(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y\right) + \Delta S_i^z S_{i+1}^z + JS\left(\hat{\boldsymbol{B}}_1 \boldsymbol{S}_2 + \hat{\boldsymbol{B}}_n(t)\boldsymbol{S}_{n-1}\right)$$

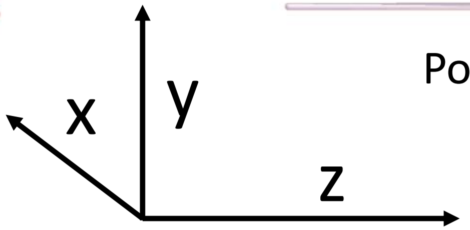
classical



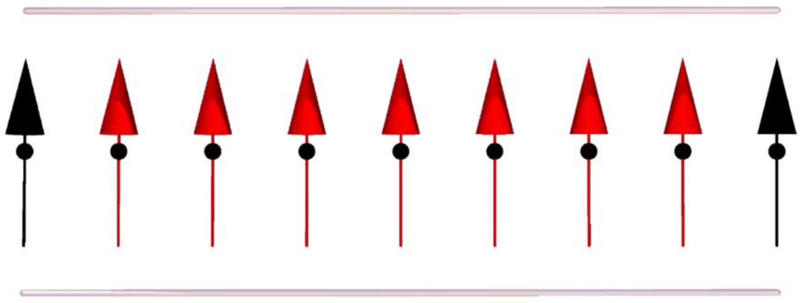
quantum



Posske, Thorwart, PRL (2019)

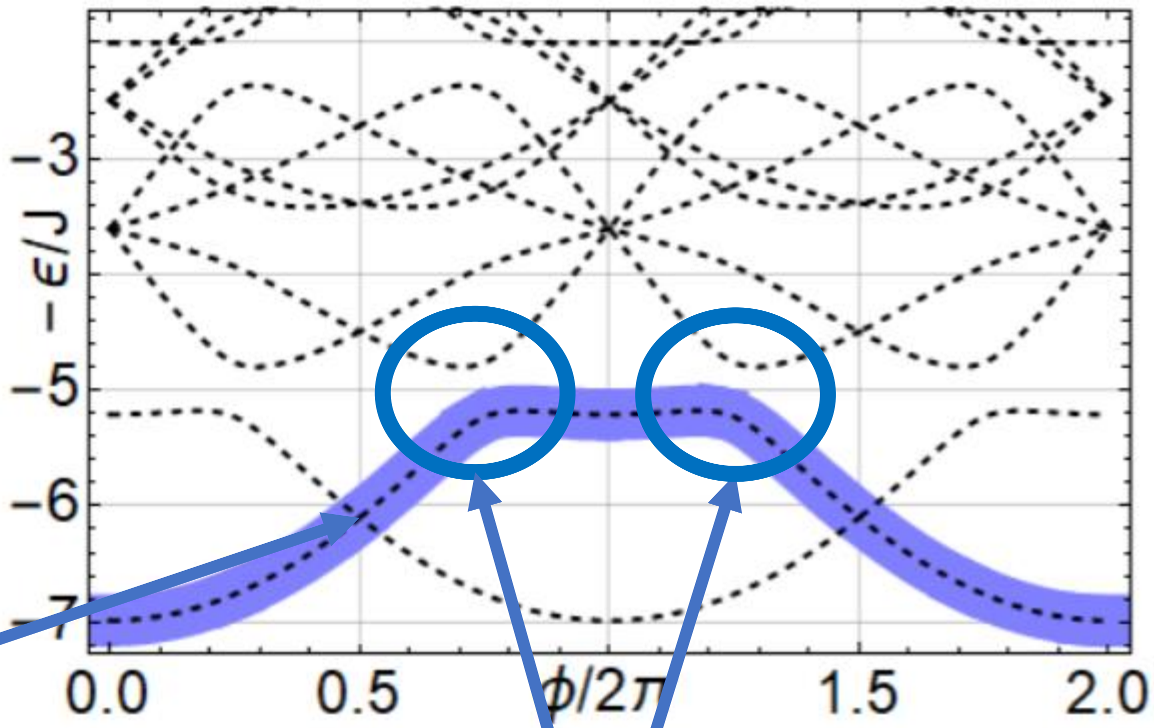


Quantum spin slippage



Adiabatic theorem

$|\psi(t+\delta t)\rangle = P_{t+\delta t}|\psi(t)\rangle$



Antiunitary symmetry & Kramers degeneracy

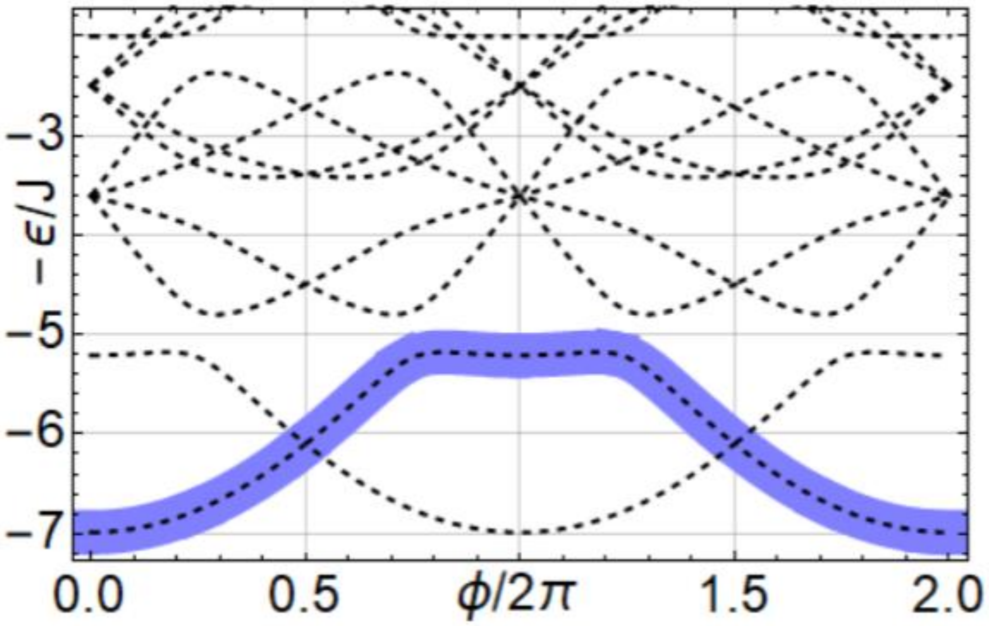
$\mathcal{A} = T_{(j\Leftrightarrow n-j+1)} e^{\mathrm{i}\sum_{k=1}^{n} S_k^y}\mathcal{K} \qquad \mathcal{A}^2 = (-1)^{2Sn}$

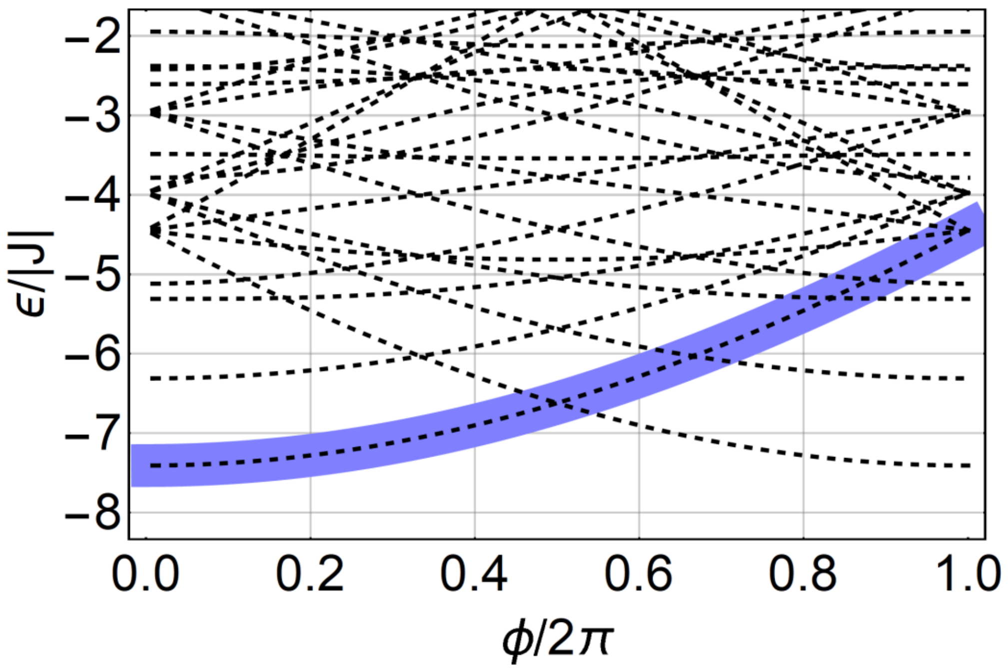
Quantum spin slippage

$|\mathrm{Singlet}\rangle \propto |\downarrow\rangle - |\uparrow\rangle$

Level crossing at sweet spots

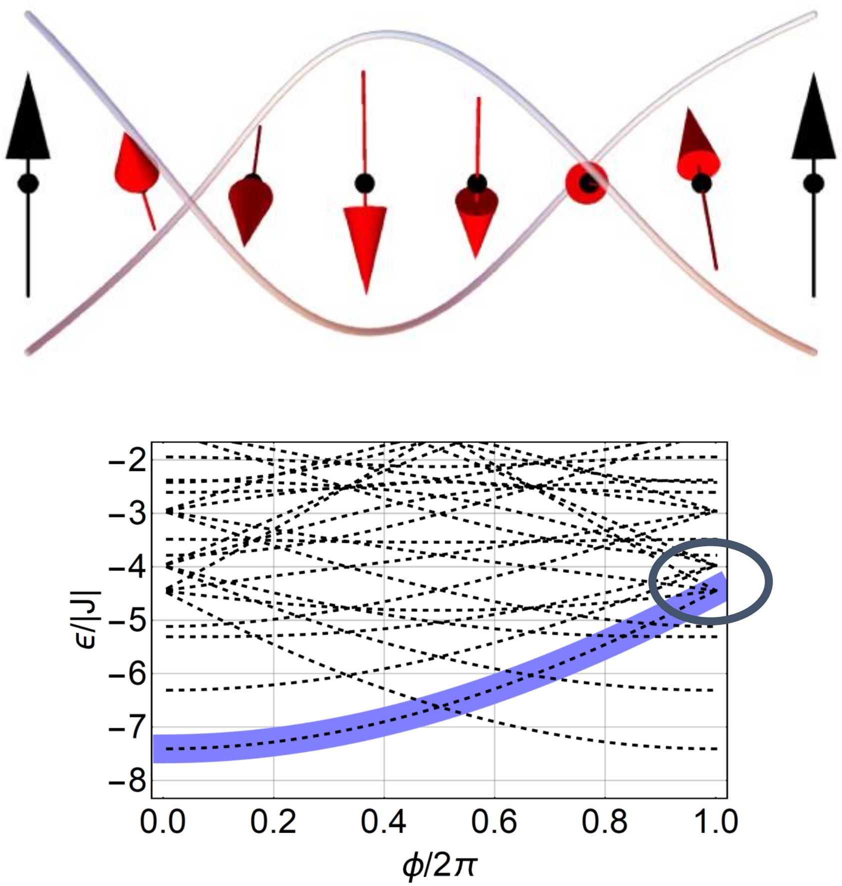
cat.	$S/\hbar$	Sweet spots Δ (in units of J)
$C1$	1/2	$\cos(\pi/m)$ with odd $m \le n-1$
	1	$\cos(\pi/m)$ with $m = n-1$
$C2$	3/2	-1.00000 -0.0483412 0.175242 0.399895 0.604953
	2	0.146900 0.278657 0.413477 0.497881



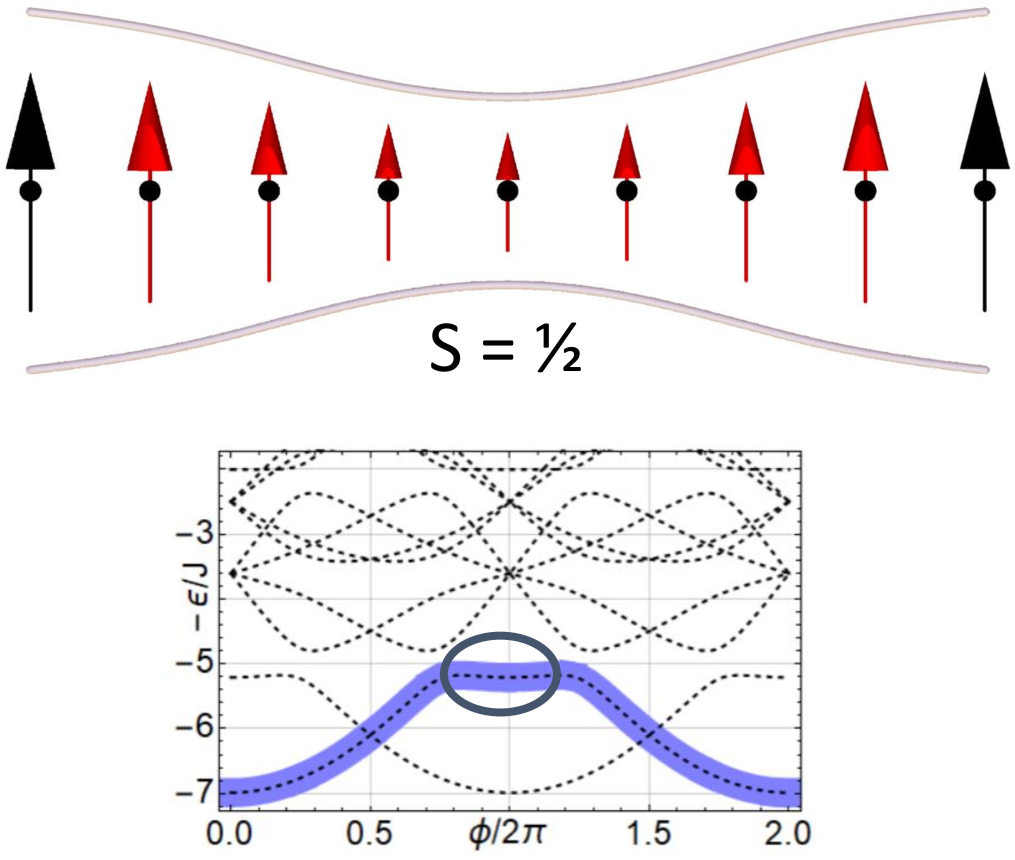


Adiabatic quasiparticle pump

Helicon

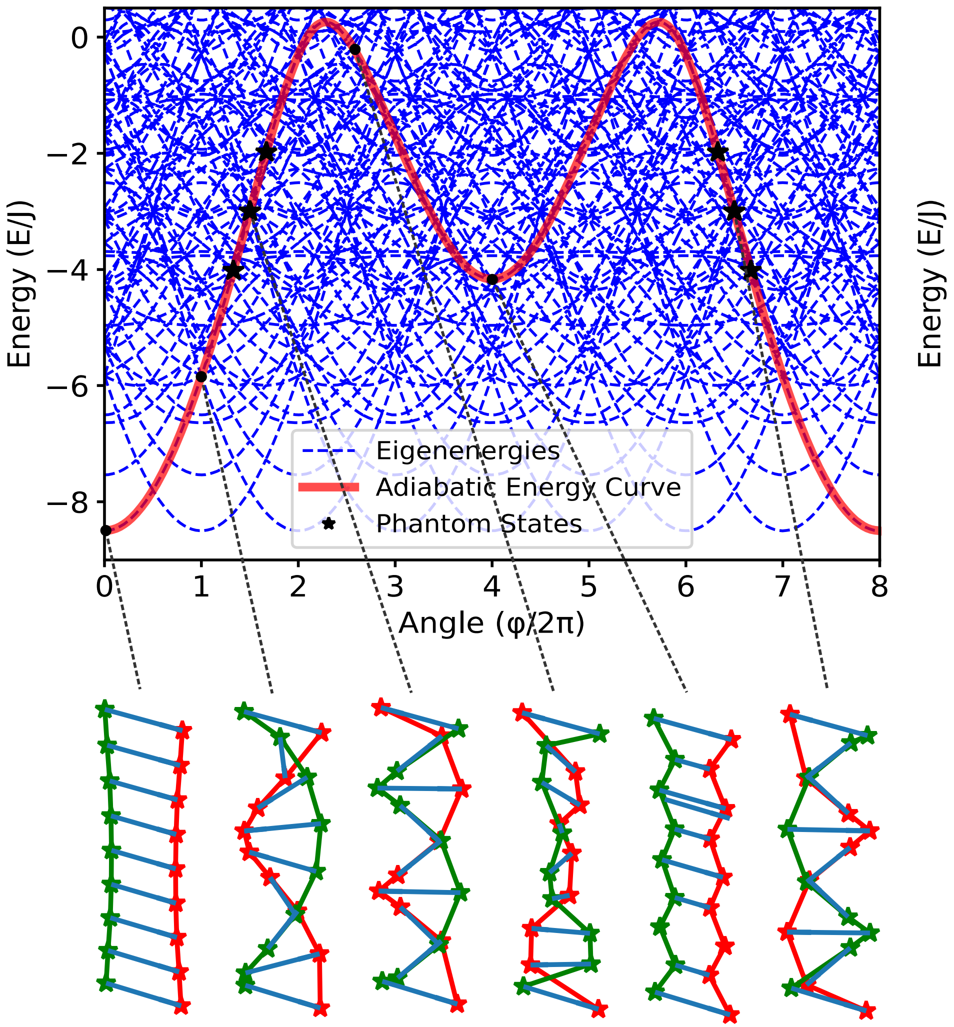


Twiston

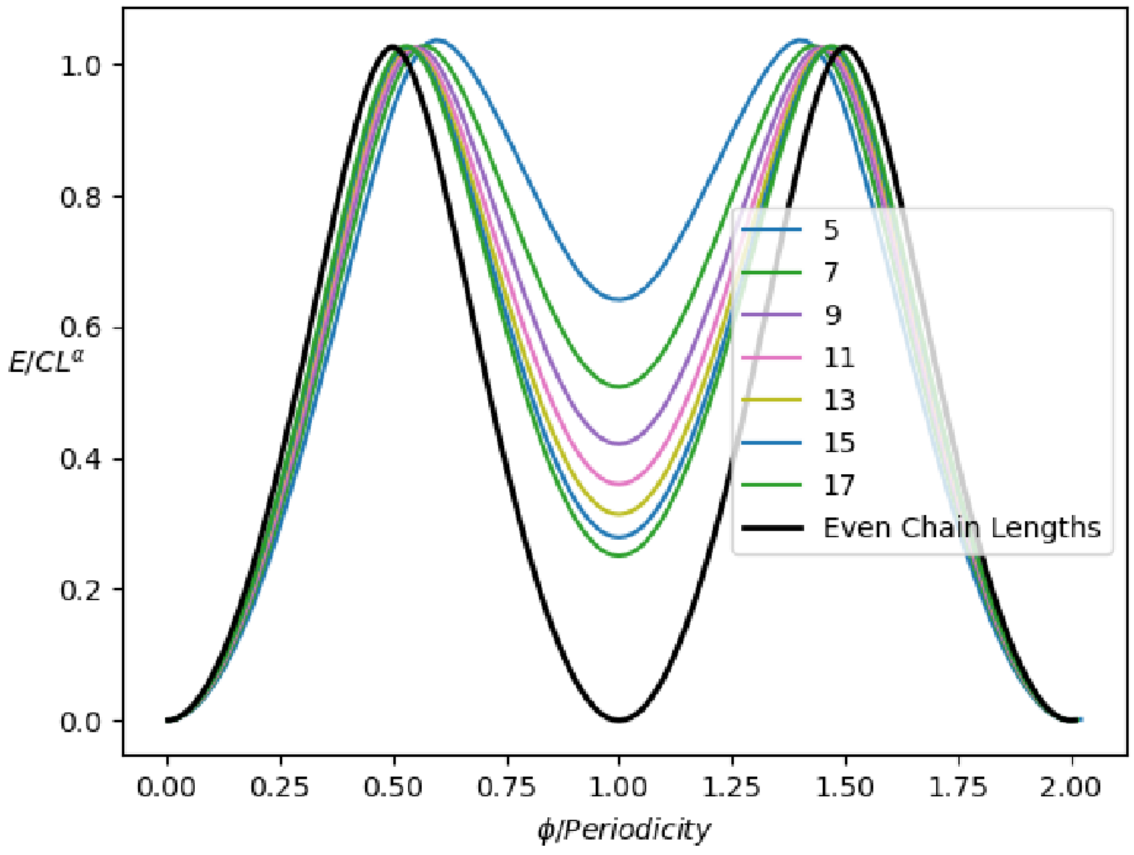


Thouless, PRB **27**, 6083 (1983); Fu, Kane PRB **74**, 195312 (2006)

Z_n effect

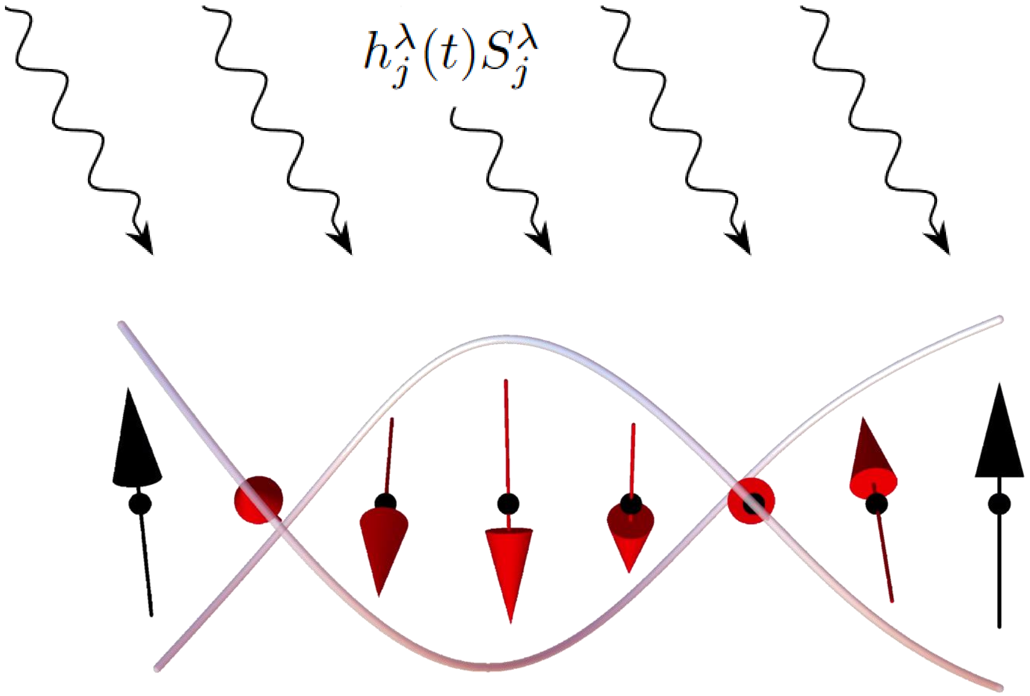


Asymptotic universal scaling



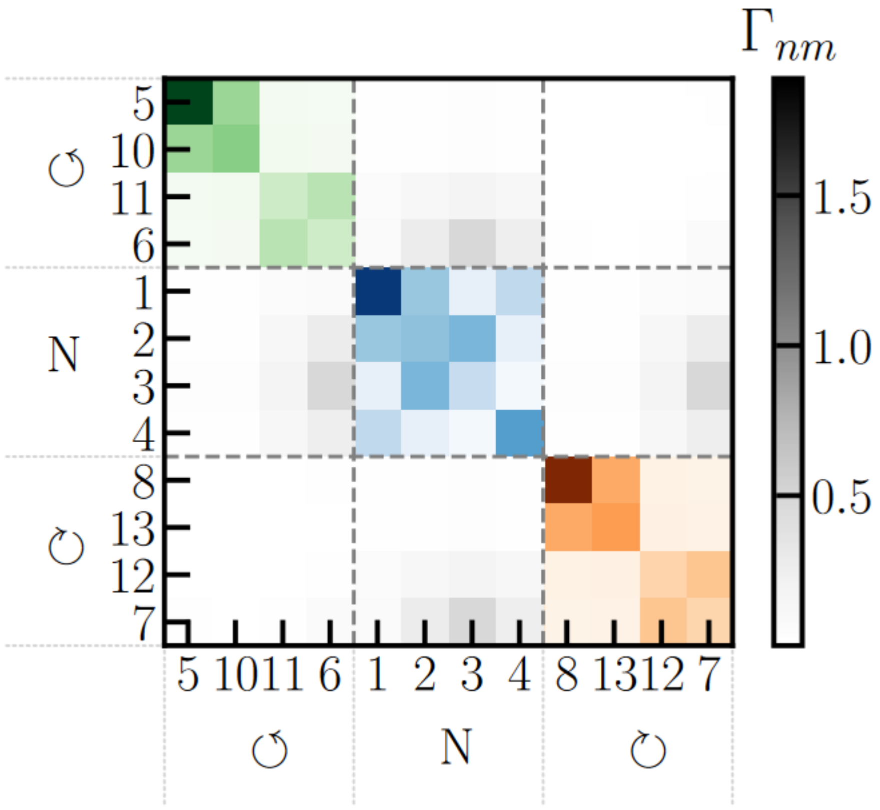
Chain Length (N)	4	5	6	7	8	9	10	11	12
Periodicity (P)	2	4	3	6	4	8	5	10	6

Stability



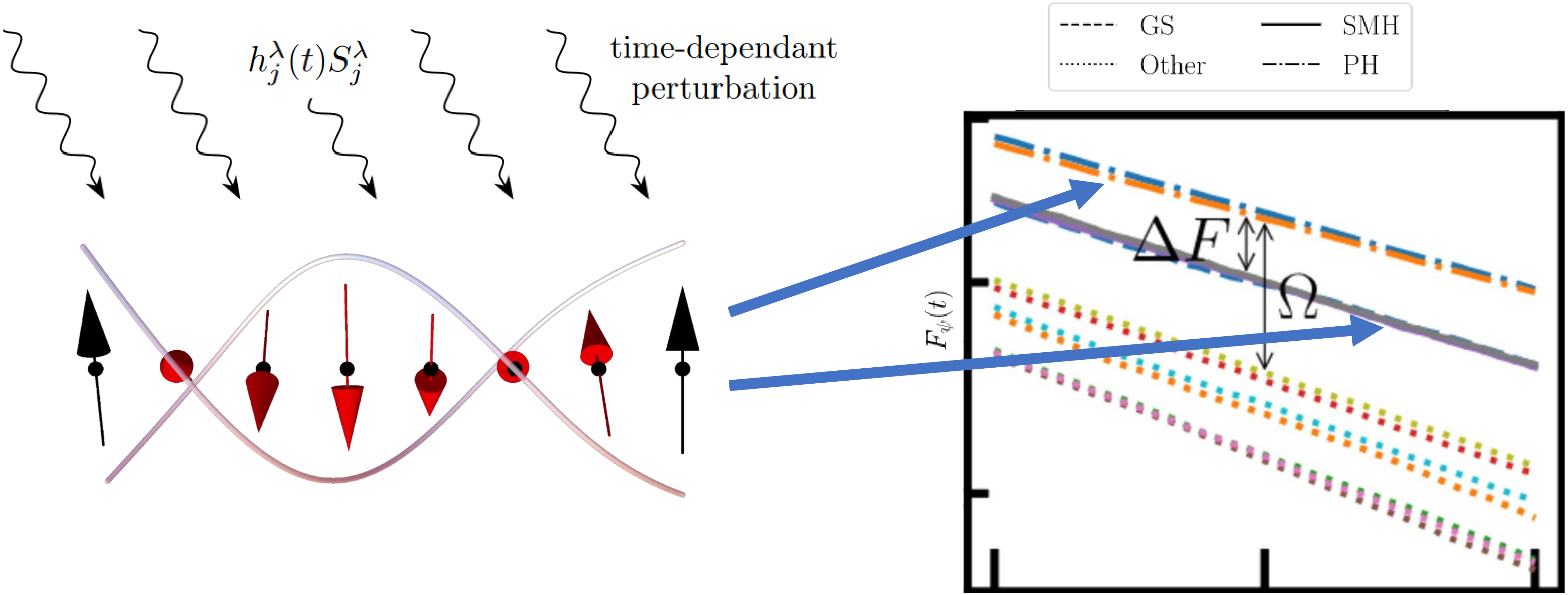
$$\Gamma_{nm} \propto \left| \left\langle n \left| \sum h_j^\lambda S_j^\lambda \right| m \right\rangle \right|^2$$

Tunneling amplitude



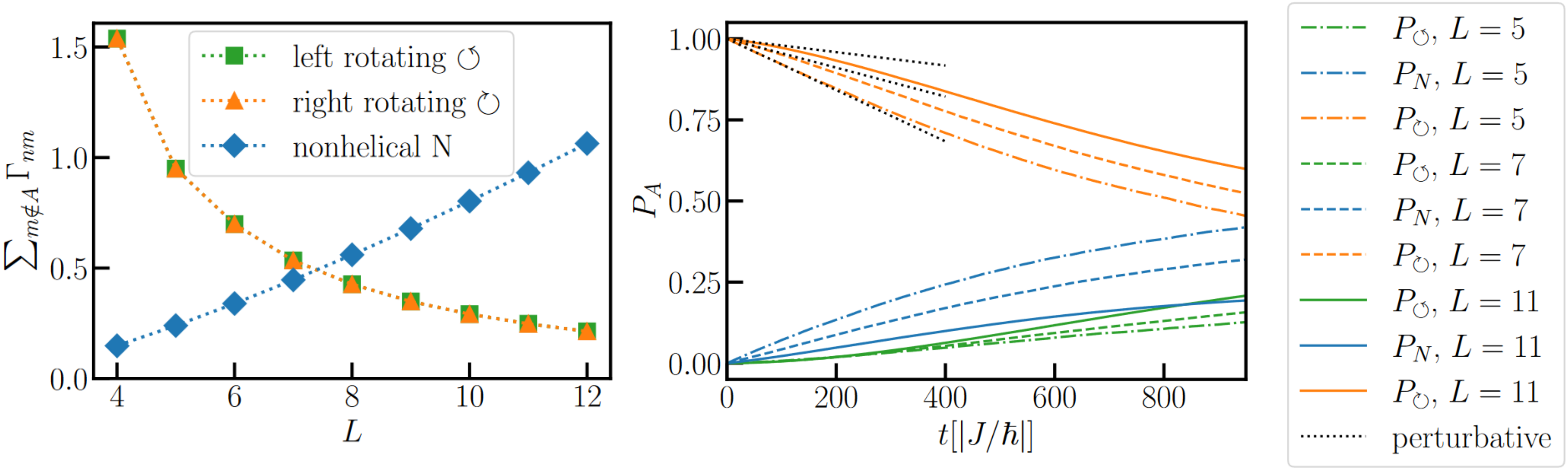
Kuehn, Gerken, .., Posske PRB (2023)

Stability

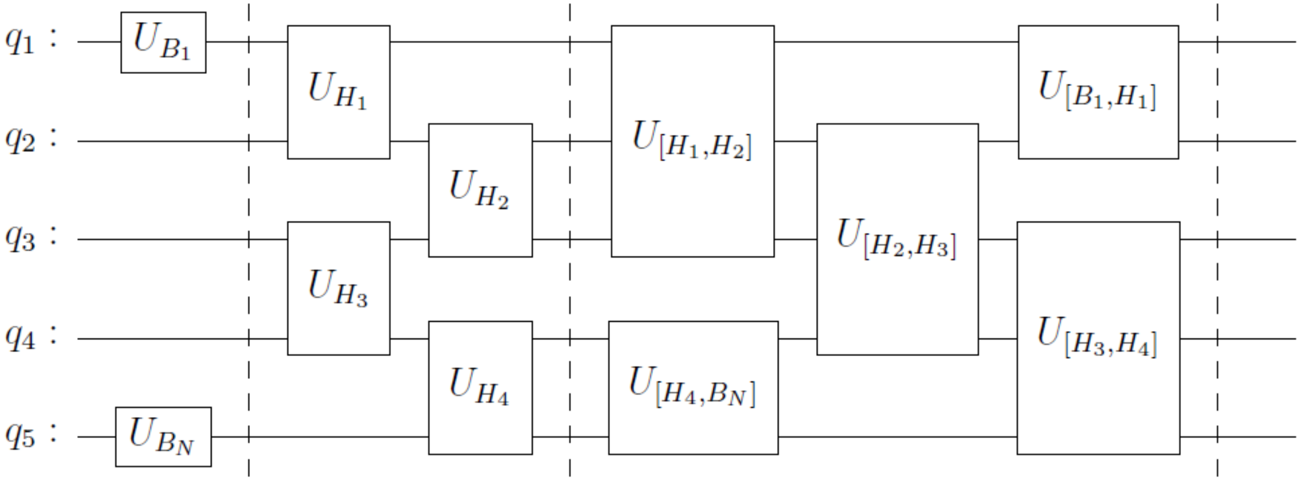


Kuehn, .., Posske, PRB (2023)

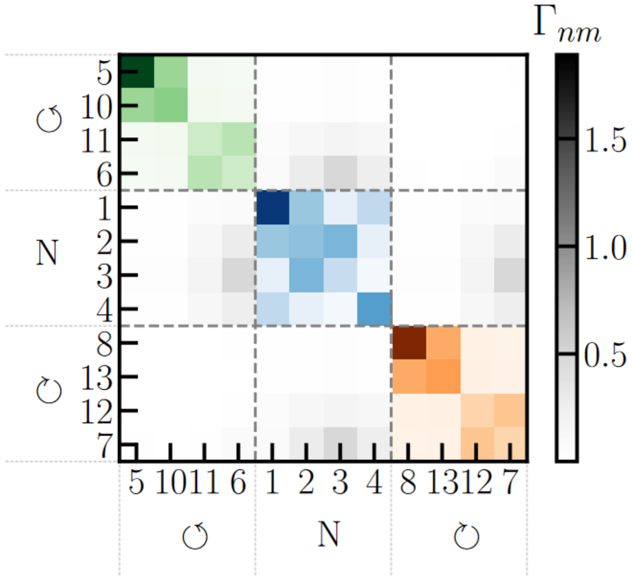
Helical sectors stability



Helices on a quantum computer



Digital quantum simulator for helical protection

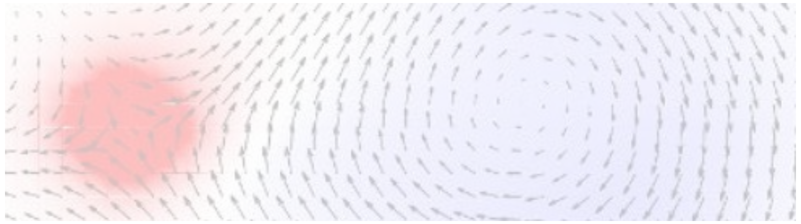


Helical sectors projective computing

Recently realized by Google team, looking for Kardar-Parisi-Zhang universality classes
Science **384**, 6691 (2024)

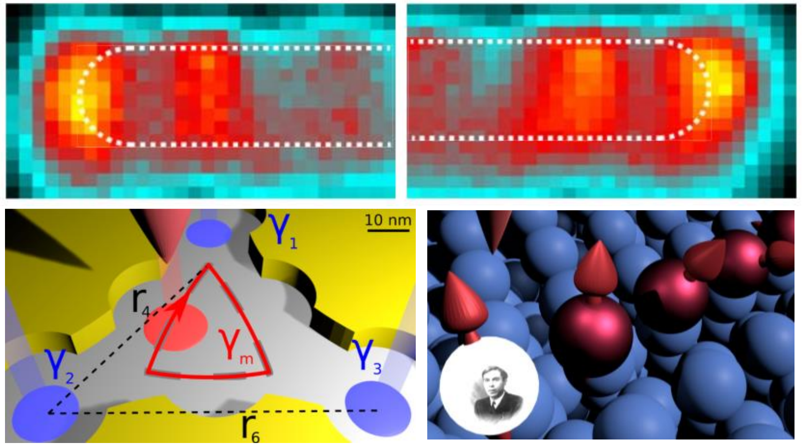
Recent research

Topological magnetism



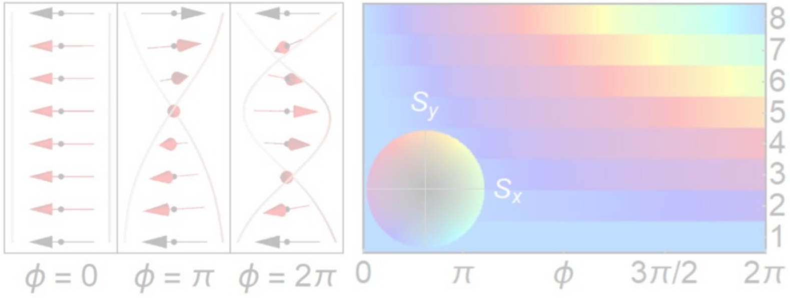
PRL (2017)

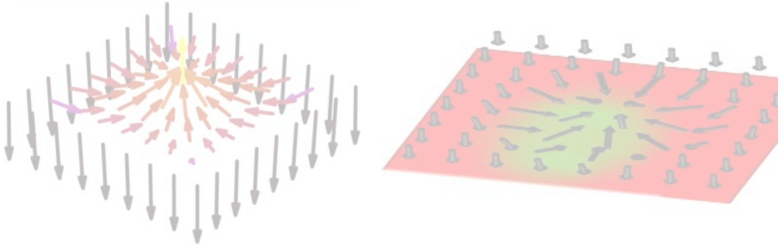
Adatom systems



Science Adv. (2018), **Nat. Phys.** (2021), **Nat. Nano.** (2022), **Nature** (2023), **US patent** (2024)

Topological quantum magnetism





PRL (2019), **DFG** (2020), **ERC** (2023)

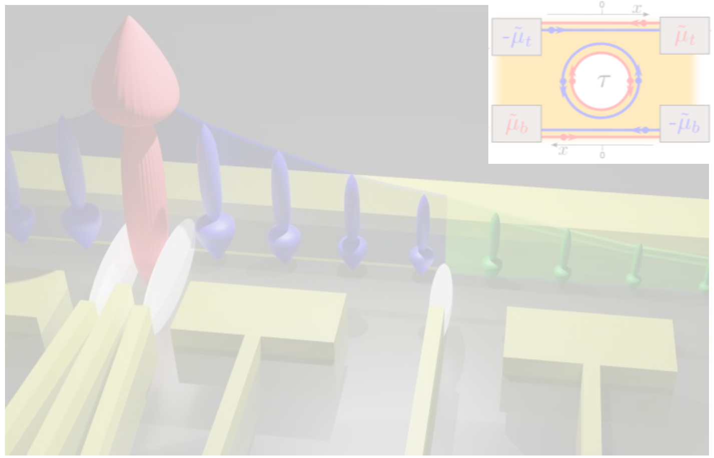
Anyons

$$a_p^\dagger a_q^\dagger = e^{i\phi_\eta(p-q)} a_q^\dagger a_p^\dagger,$$

$$a_p a_q^\dagger = e^{-i\phi_\eta(p-q)} a_q^\dagger a_p + \delta(p-q)$$

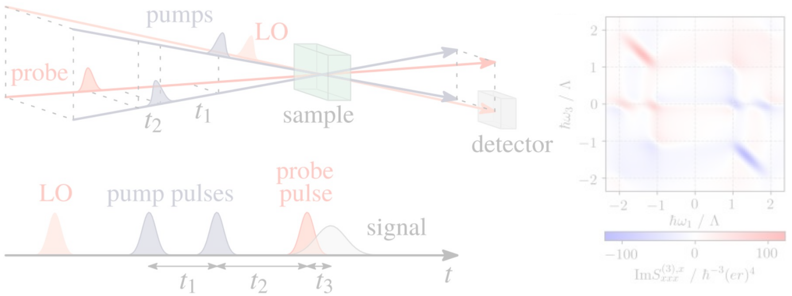
PRL (2021)

Kondo effect



PRL (2013), **PRL** (2015)

Spectroscopy + top. matter



PRL (2022)

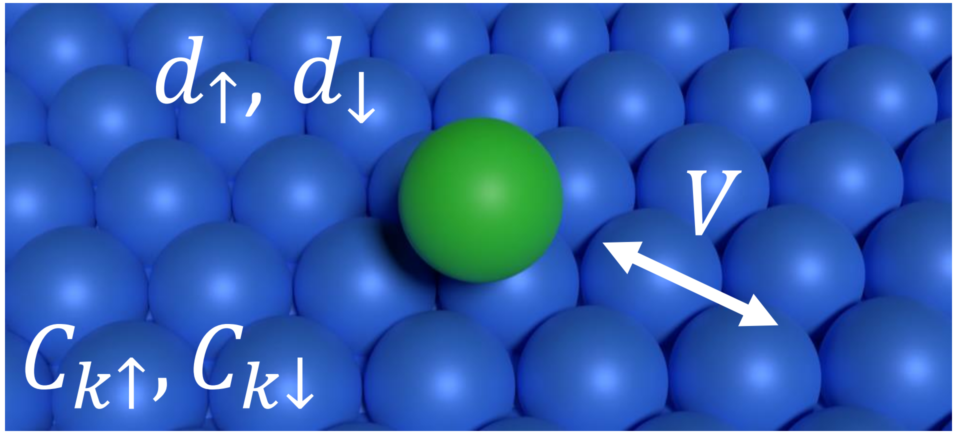
Proximity superconductivity in atom-by-atom crafted quantum dots: Machida Shibata states

Lucas Schneider, Khai That Ton, Ioannis Ioannidis, Jannis Neuhaus-Steinmetz, Thore Posske, Roland Wiesendanger & Jens Wiebe

Nature **621**, 60–65 (2023) | Cite this article

In-gap state from nonmagnetic adatoms?

Kazushige Machida, Fumiaki Shibata (1972)



$$\mathscr{H}=\sum_{k\sigma}\epsilon_k C^\dagger_{k\sigma}C_{k\sigma}-\Delta\sum_{k}(C^\dagger_{k\uparrow}C^\dagger_{-k\downarrow}+C_{-k\downarrow}C_{k\uparrow})$$
$$+\sum_{k\sigma}V_{kd}(C^\dagger_{k\sigma}d_\sigma+d_\sigma^\dagger C_{k\sigma})+E\sum_{\sigma}n_{d\sigma}$$

Progress of Theoretical Physics, Vol. 47, No. 6, June 1972

Bound States Due to Resonance Scattering in Superconductor

Kazushige MACHIDA and Fumiaki SHIBATA

Department of Physics, Tokyo University of Education, Tokyo

(Received December 27, 1971)

It is shown exactly that the bound state in the energy gap of superconductors is produced by the resonance scattering due to a single non-magnetic impurity.

In an appropriate renormalization procedure, we can show the growth of an impurity band in the case of impurities of finite concentration and a possibility for gapless superconductivity is indicated.

$$\omega^2\left(1+\frac{2\Gamma}{\sqrt{\Delta^2-\omega^2}}\right)=E^2+\Gamma^2$$

$$\Gamma = V^2\,\pi \times SC\ Density\ of\ states$$

Where is the in-gap state?

Hybridization Γ much larger than superconducting gap Δ: $\Gamma \approx 100 - 1000\,\Delta$

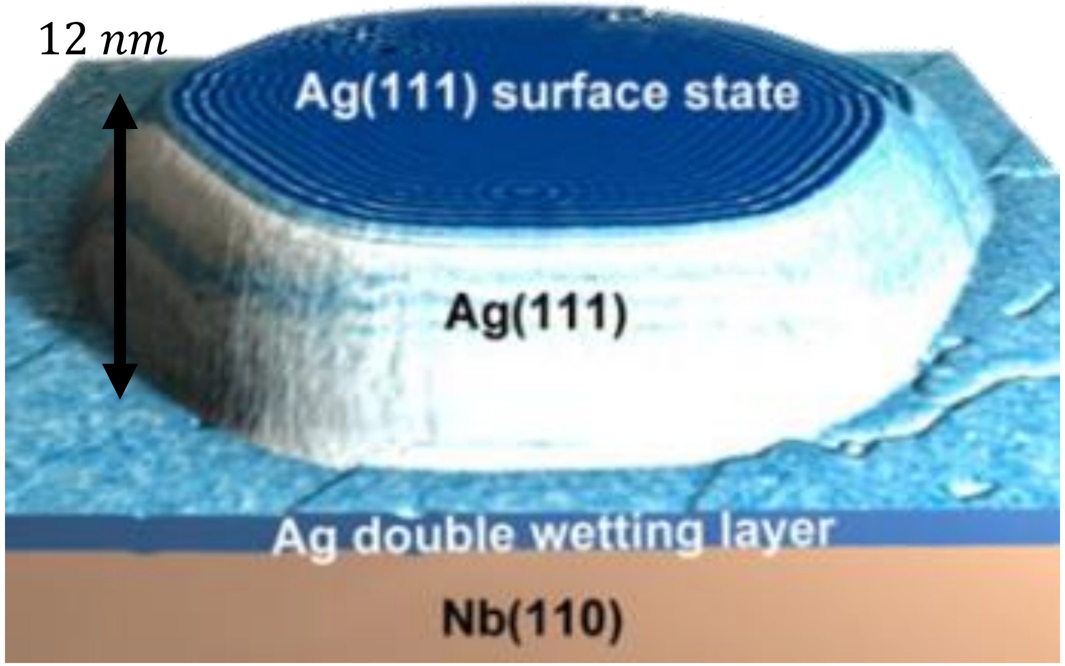
$$\omega^2\left(\cancel{1}+\frac{2\cancel{\Gamma}}{\sqrt{\Delta^2-\omega^2}}\right)=\cancel{E^2}+\Gamma^{\cancel{2}}$$

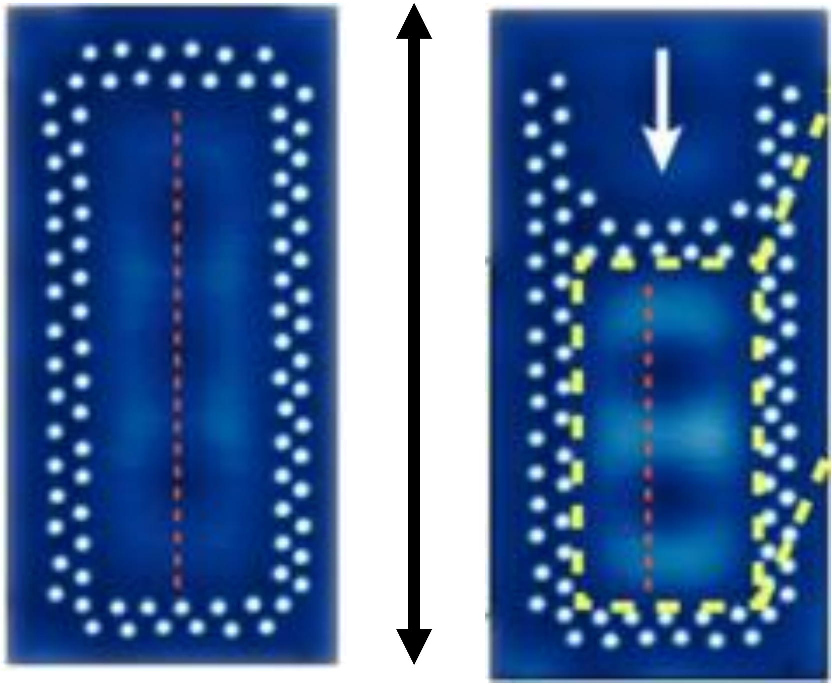
$$\omega \approx \Delta$$

“ In most physical situations [...] the bound state lies essentially at the gap edge. [...] Shiba [...] concluded [... that they] can be neglected in discussion of physical properties.

A. V. Balatsky, I. Vekhter, and Jian-Xin Zhu
Rev. Mod. Phys. **78**, 373 (2006)

Experimental setup for Machida-Shibata states

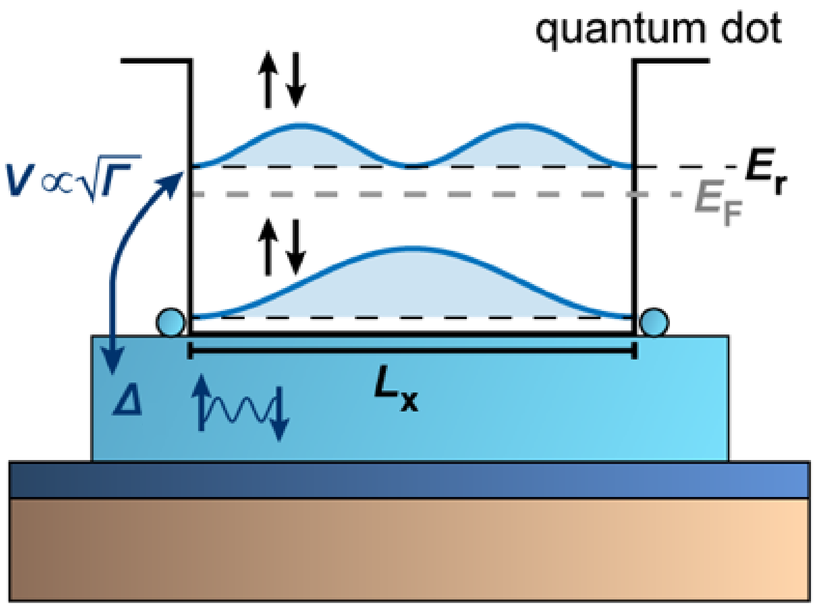




$\Delta_{Nb} = 1.50\ meV$
$\Delta = 1.35\ meV$

Schneider et al., Nature **621,** 60 (2023)

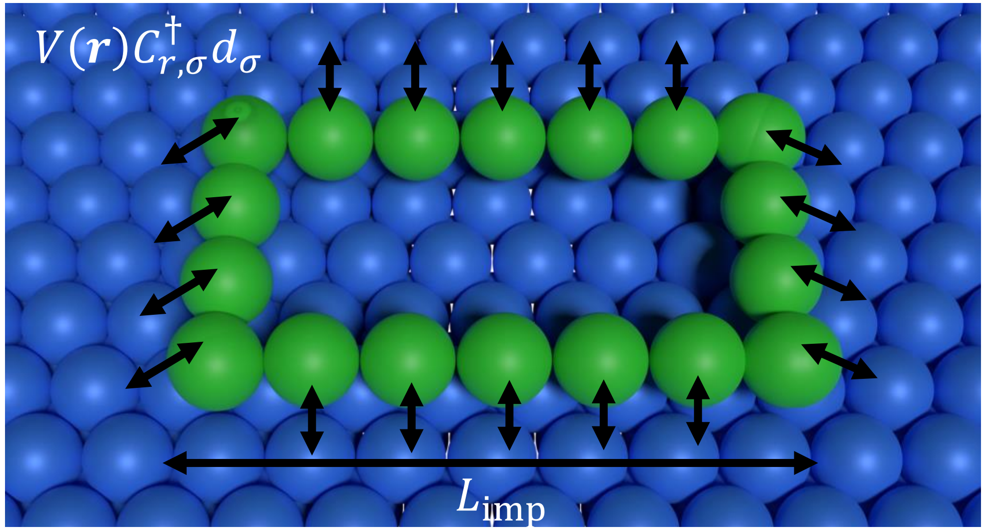
Experimental setup for Machida-Shibata states



1) Particle-in-a-box state couples to superconductor only close to boundary
2) Capacitive screening by superconductor
3) Energy tunable by quantum dot length
   $$E \propto n^2/L^2$$

Schneider et al., Nature **621,** 60 (2023)

Model of extended impurity



$$\mathcal{H}_{coupling} = \sum_{\mathbf{k},\sigma} V(\mathbf{k}) c^{\dagger}_{\mathbf{k},\sigma} d_{\sigma} + h.c.$$

$$\mathcal{H}_0 = \sum_{k,\sigma} \epsilon_k c^{\dagger}_{k,\sigma} c_{k,\sigma} + \sum_{\sigma} E_{\mathrm{r}} d^{\dagger}_{\sigma} d_{\sigma}$$

$$\mathcal{H}_{SC} = -\Delta_{\mathrm{s}} \sum_{k} \left(c^{\dagger}_{k,\uparrow} c^{\dagger}_{-k,\downarrow} + c_{-k,\downarrow} c_{k,\uparrow} \right)$$

Solve Green's functions equation of motions

$$G_{d_{\sigma},d^{\dagger}_{\sigma}}(\omega) = \frac{\omega + E_{\mathrm{r}} + \sum_{\mathbf{k}} |V(\mathbf{k})|^2 \frac{(\omega - \epsilon_{\mathbf{k}})}{(\omega^2 - \epsilon_{\mathbf{k}}{}^2 - \Delta_{\mathrm{s}}{}^2)}}{\left(\omega + E_{\mathrm{r}} - \sum_{\mathbf{k}} \frac{|V(\mathbf{k})|^2 (\omega - \epsilon_{\mathbf{k}})}{(\omega^2 - \epsilon_{\mathbf{k}}{}^2 - \Delta_{\mathrm{s}}{}^2)}\right)\left(\omega - E_{\mathrm{r}} - \sum_{\mathbf{k}} \frac{|V(\mathbf{k})|^2 (\omega + \epsilon_{\mathbf{k}})}{(\omega^2 - \epsilon_{\mathbf{k}}{}^2 - \Delta_{\mathrm{s}}{}^2)}\right) - \left(\sum_{\mathbf{k}} \frac{\Delta_s V(\mathbf{k})^2}{(\omega^2 - \epsilon_{\mathbf{k}}{}^2 - \Delta_{\mathrm{s}}{}^2)}\right)\left(\sum_{\mathbf{k}} \frac{\Delta_s V(\mathbf{k})^{*2}}{(\omega^2 - \epsilon_{\mathbf{k}}{}^2 - \Delta_{\mathrm{s}}{}^2)}\right)}$$

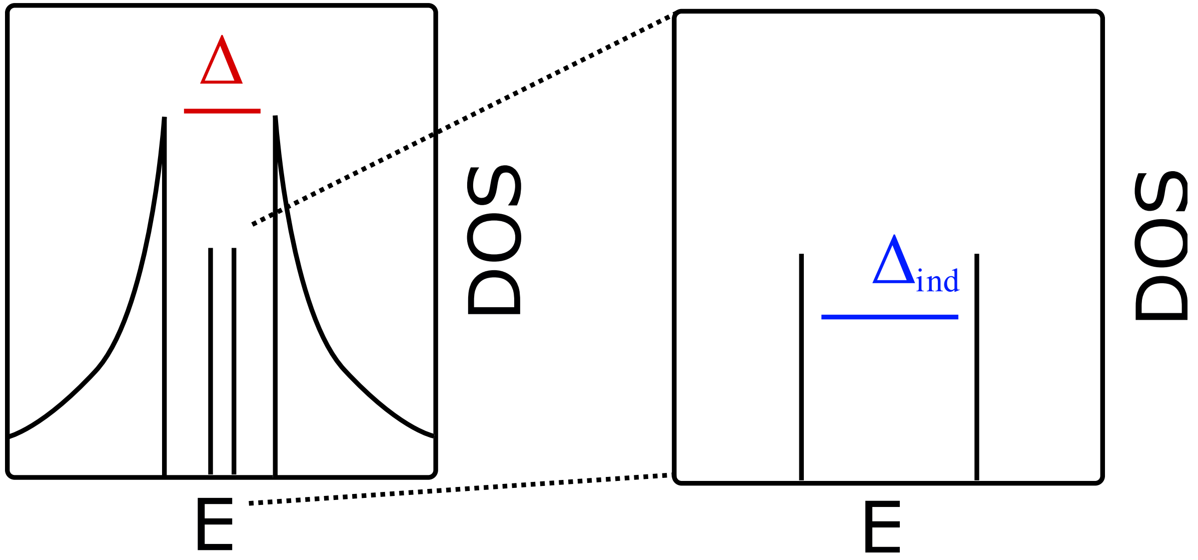
Like single level superconductivity

$$\mathcal{H} = \sum_{k,\sigma} \epsilon_k c^\dagger_{k,\sigma} c_{k,\sigma} + \sum_{k,\sigma} V\left(c^\dagger_{k,\sigma} d_\sigma + d^\dagger_\sigma c_{k,\sigma}\right) + \sum_{\sigma} E_0 d^\dagger_\sigma d_\sigma - \Delta_\mathrm{s} \sum_{k} \left(c^\dagger_{k,\uparrow} c^\dagger_{-k,\downarrow} + c_{-k,\downarrow} c_{k,\uparrow}\right)$$

Low energy theory
(Schrieffer-Wolff)

$$\mathcal{H}'_\mathrm{D} = \sum_\sigma (E_\mathrm{r} + E_\mathrm{shift}) d^\dagger_\sigma d_\sigma - \left(\Delta_\mathrm{ind} d^\dagger_\uparrow d^\dagger_\downarrow + \mathrm{h.c.}\right)$$

$$\Delta_\mathrm{ind} \propto V^2 \frac{\Delta_\mathrm{s}}{\sqrt{\Delta_\mathrm{s}^2 - E_0^2}} \qquad E_\mathrm{shift} = E_\mathrm{r} \frac{\Delta_\mathrm{ind}}{\Delta_\mathrm{s}}$$



MSS energies:

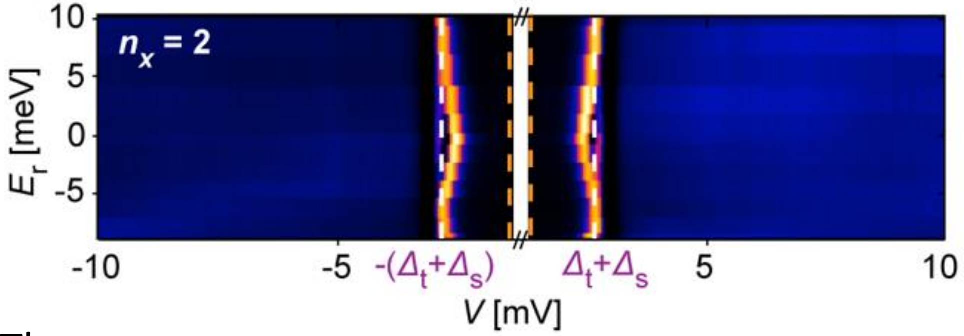
$$\varepsilon = \pm\sqrt{{E_\mathrm{r}}^2 (1 - \Delta_\mathrm{ind}/\Delta_\mathrm{s})^2 + {\Delta_\mathrm{ind}}^2}$$

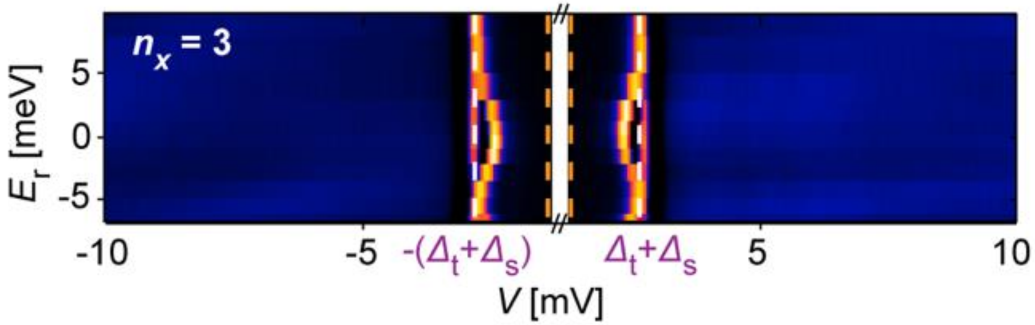
Particle-hole weight:

$$|v|^2 = \frac{1}{2} - \frac{E_\mathrm{r}\left(1 - \frac{\Delta_\mathrm{ind}}{\Delta_\mathrm{s}}\right)}{2\varepsilon}, \quad |u|^2 = 1 - |v|^2$$

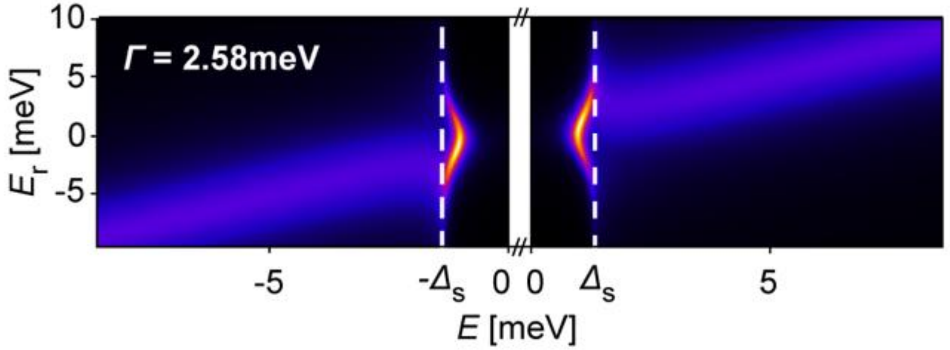
Density of states (dI/dV) Measurements

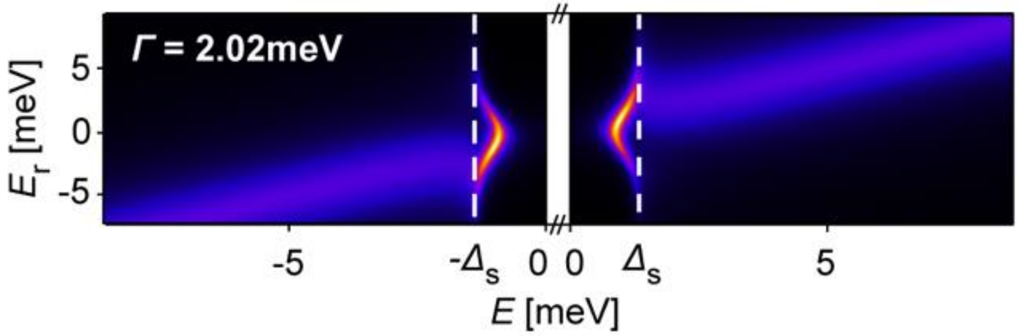
Experiment $n_x = 2$ $n_x = 3$



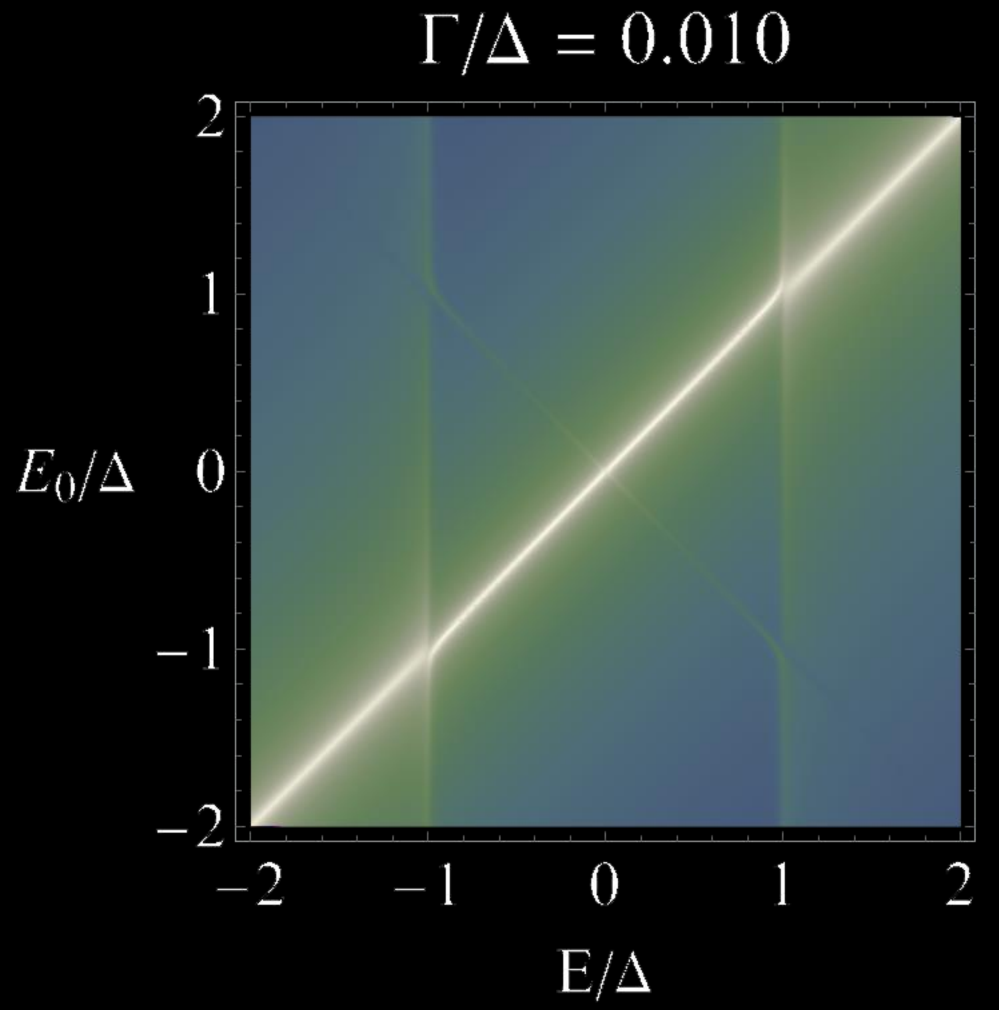


Theory

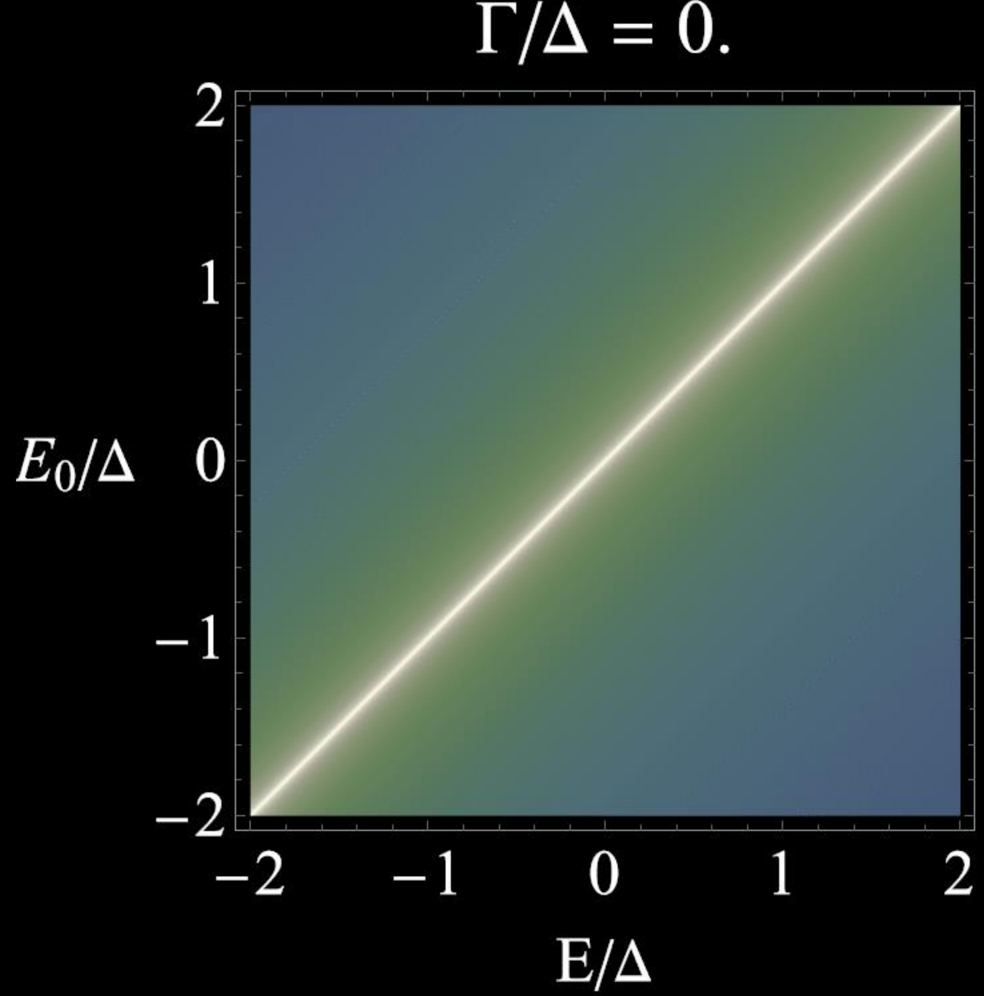




Single level superconductivity (DOS)



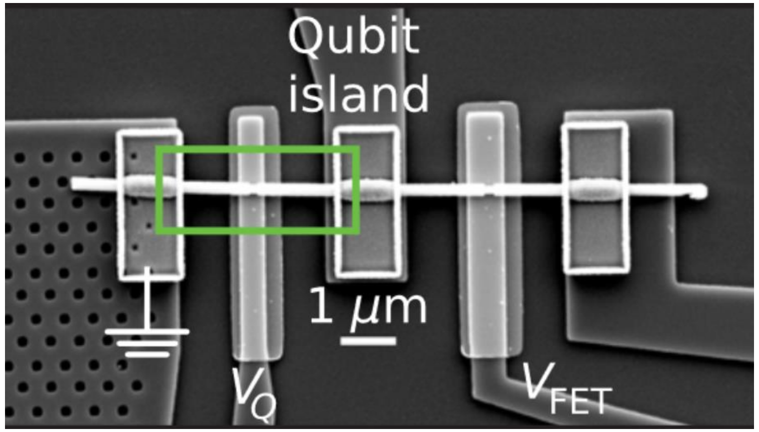
Single level superconductivity (DOS)



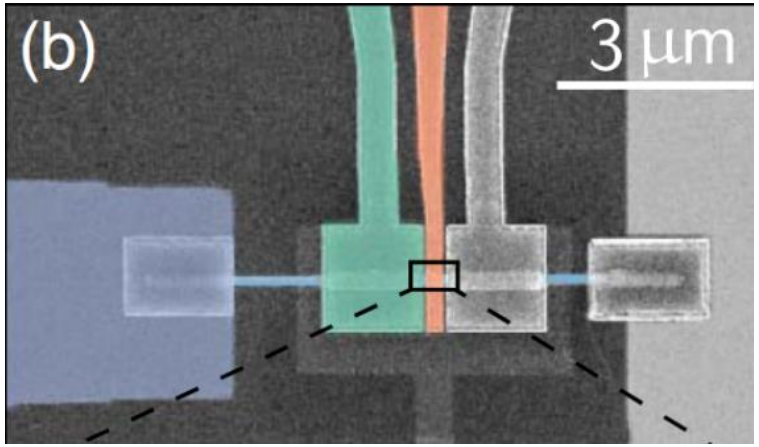
Machida-Shibata states in transmon/gatemon qubits

Kazushige Machida

“I thought for long time that transition metal non-magnetic impurities produce the in-gap state, but the location of it is so near the superconducting gap edge, thus it is impossible to prove its existence. But by your ingenious method you have finally checked it to be true experimentally.
[…]
I hope that this time reversal preserving Machida-Shibata state turns out to be of some use for the future as application or fundamental physics.”



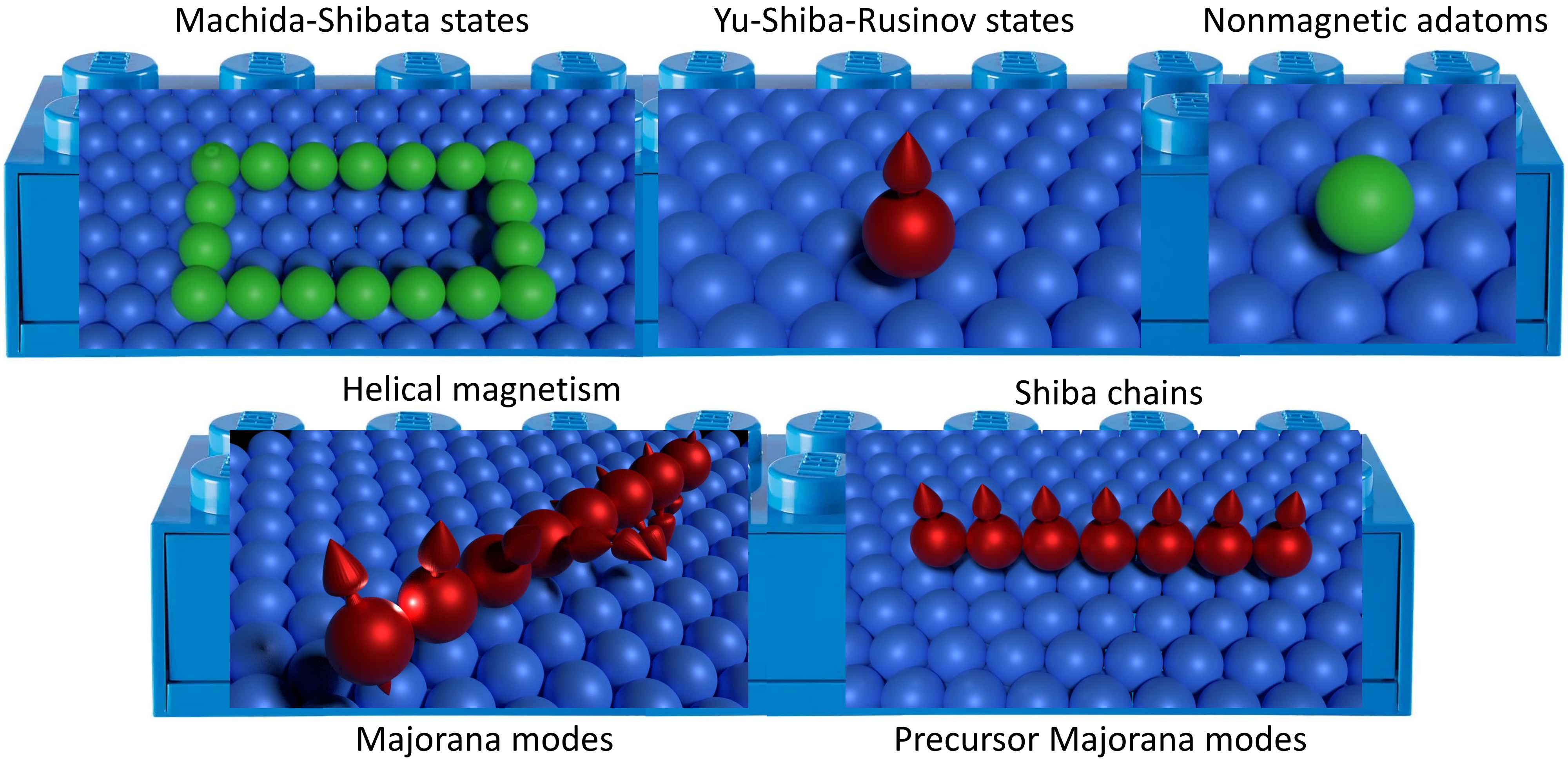
Kringhøj et al. PRL **124**, 246803 (2020)



Bargerbos et al. PRL **124**, 246802 (2020)

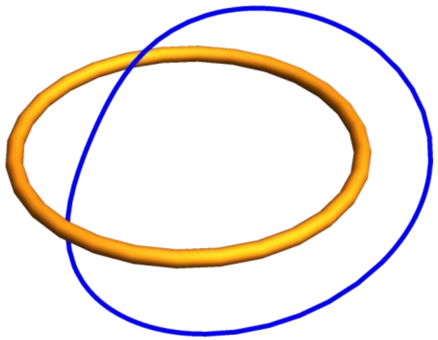
Suppressed voltage noise + strong anharmonicity

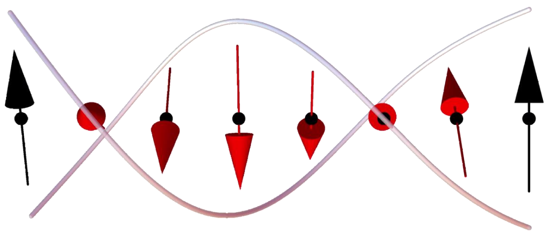
Lego kit of in-gap states for synthesizing 2D quantum matter

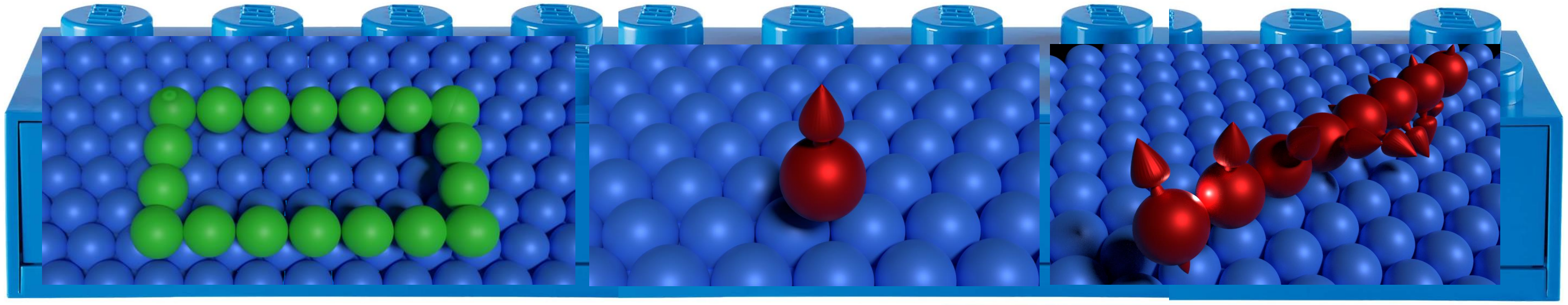


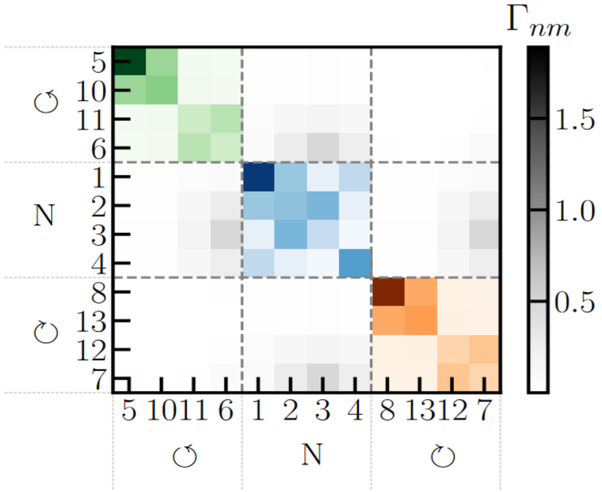
Summary

- Topological quantum magnetism hosts exotic quasiparticles and stable helical sectors
- Adatom systems on superconductors are versatile building blocks for 2D in-gap quantum matter

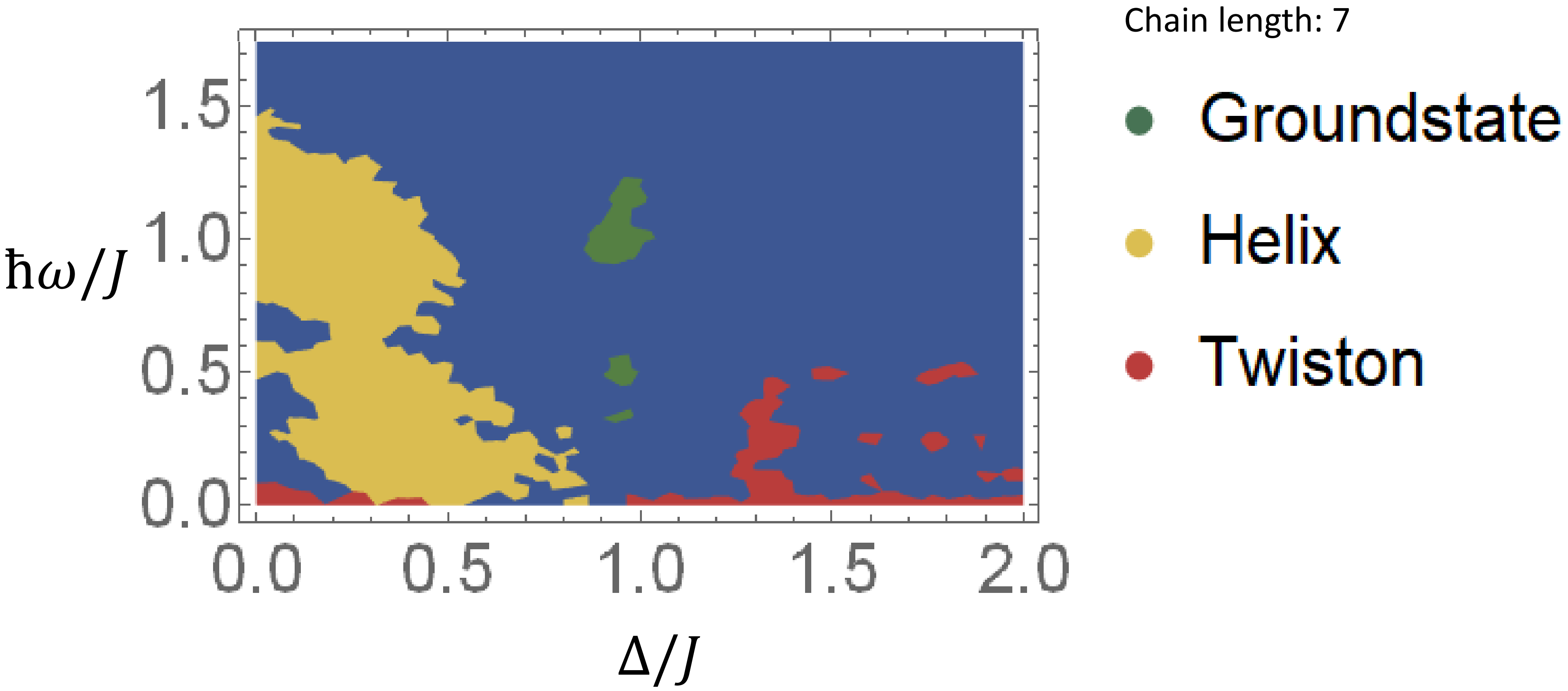






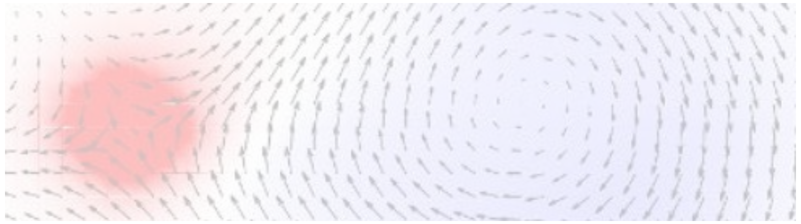


Dynamical creation of helices



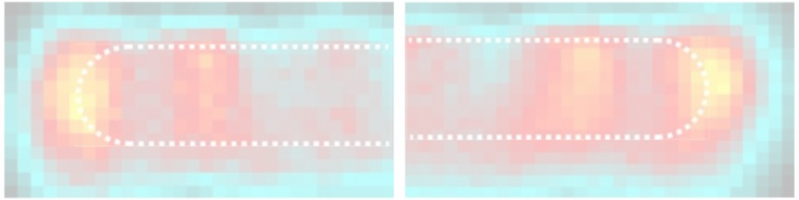
Recent research

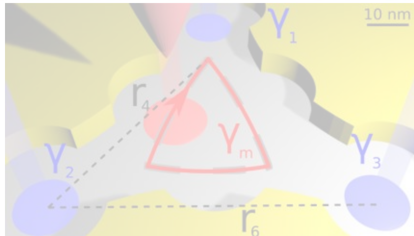
Topological magnetism

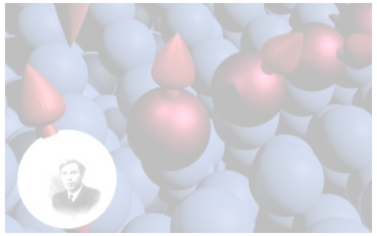


PRL (2017)

Topological phases & Majoranas

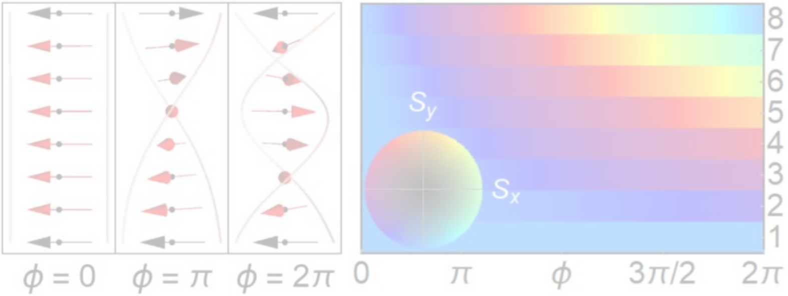


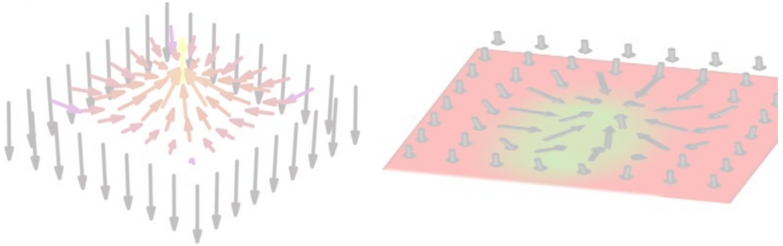




Science Adv. (2018), **Nat. Phys.** (2021), **Nat. Nano.** (2022), **Nature** (2023), **US patent** (2021)

Topological quantum magnetism





PRL (2019), **DFG** (2020), **ERC** (2023)

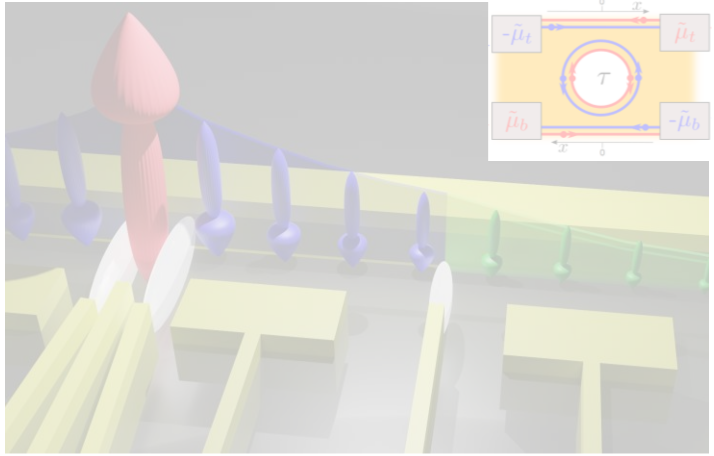
Anyons

$$a_p^\dagger a_q^\dagger = e^{i\phi_\eta(p-q)} a_q^\dagger a_p^\dagger,$$

$$a_p a_q^\dagger = e^{-i\phi_\eta(p-q)} a_q^\dagger a_p + \delta(p-q)$$

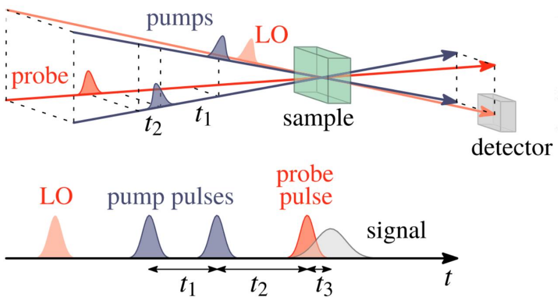
PRL (2021)

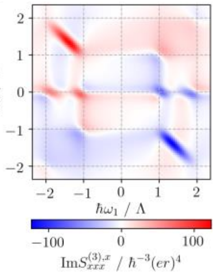
Kondo effect



PRL (2013), **PRL** (2015)

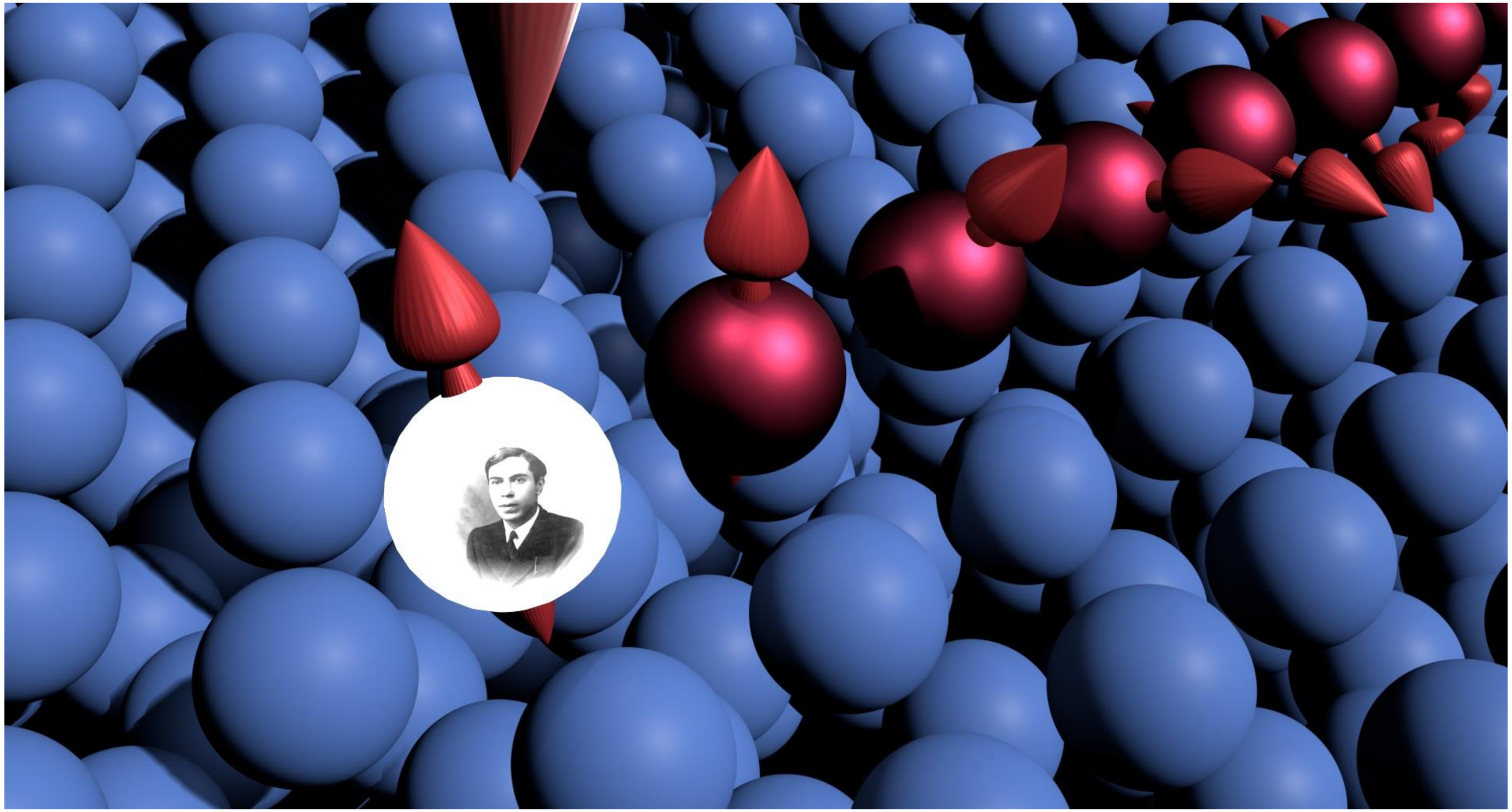
Spectroscopy + top. matter





PRL (2022)

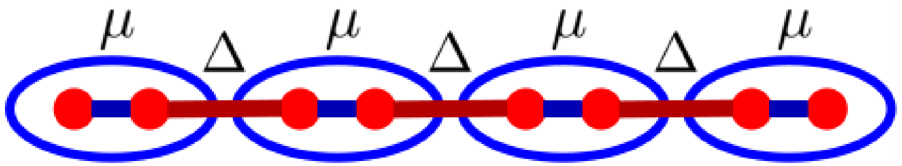
Multidimensional spectroscopy of topological superconductors

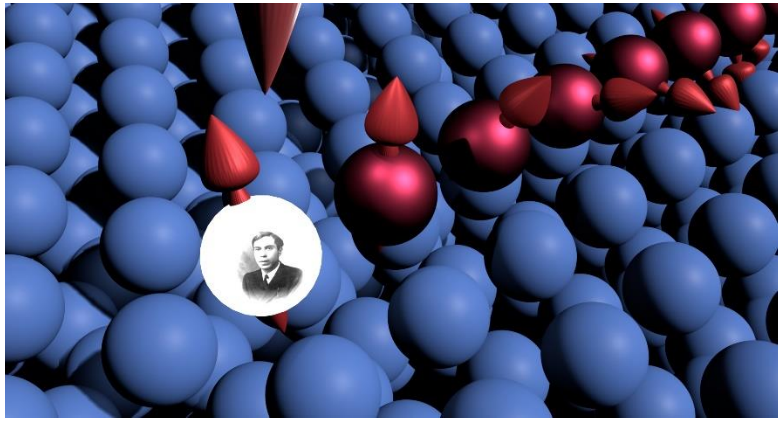


One-dimensional topological superconductivity

μ chemical potential

Δ superconductivity and hopping





$$H = \sum_{n=1}^{N} \left[-w a_{n+1}^{\dagger} a_n - \mu a_n^{\dagger} a_n + \Delta a_n a_{n+1} \right] + \text{h.c.}$$

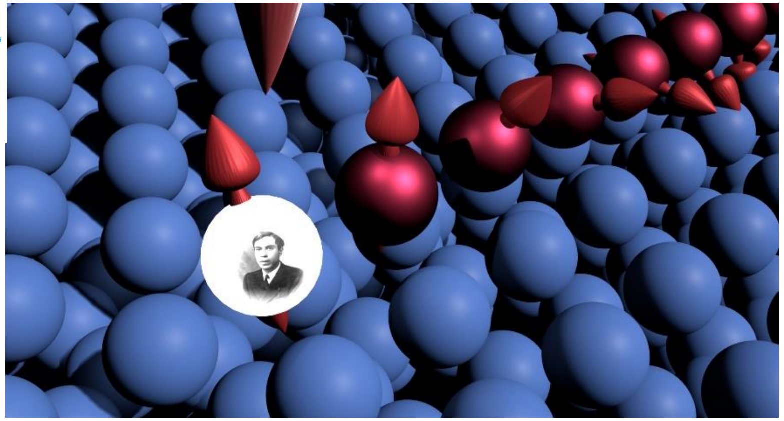
real fermions:
Majorana modes

$\gamma_{j,1} = a_j + a_j^{\dagger}$ $\qquad a_j = (\gamma_{j,1} + i\gamma_{j,2})/2$

$\gamma_{j,2} = (a_j - a_j^{\dagger})/i$ $\qquad a_j^{\dagger} = (\gamma_{j,1} - i\gamma_{j,2})/2$

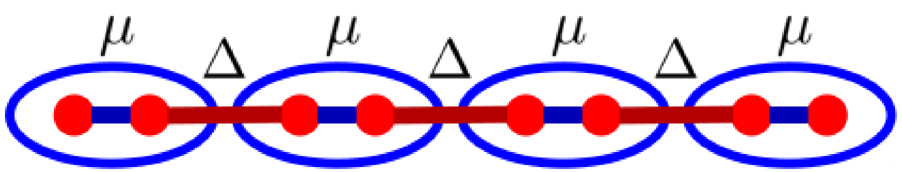
One-dimensional topological superconductivity

$$H = \sum_{n=1}^{N} \left[-w a_{n+1}^{\dagger} a_n - \mu a_n^{\dagger} a_n + \Delta a_n a_{n+1} \right] + \text{h.c.}$$

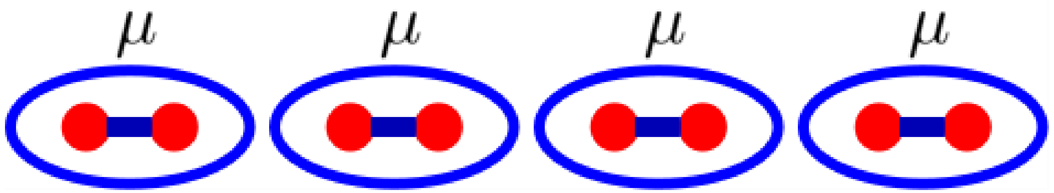


μ chemical potential

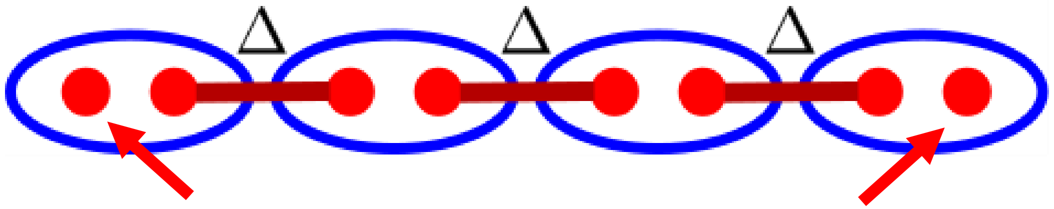
Δ superconductivity and hopping



fermionic phase
$\Delta = 0$



Majorana phase
$\mu = 0$



free Majoranas

One-dimensional topological superconductivity

- Andreev bound states
- Yu-Shiba-Rusinov states
- Caroli-de-Gennes-Matricon states
- Kondo resonance
- particle-in-a-box state
- Andersson localization

Proposal:
(1) exclude boundary modes
(2) exclude other spectral differences

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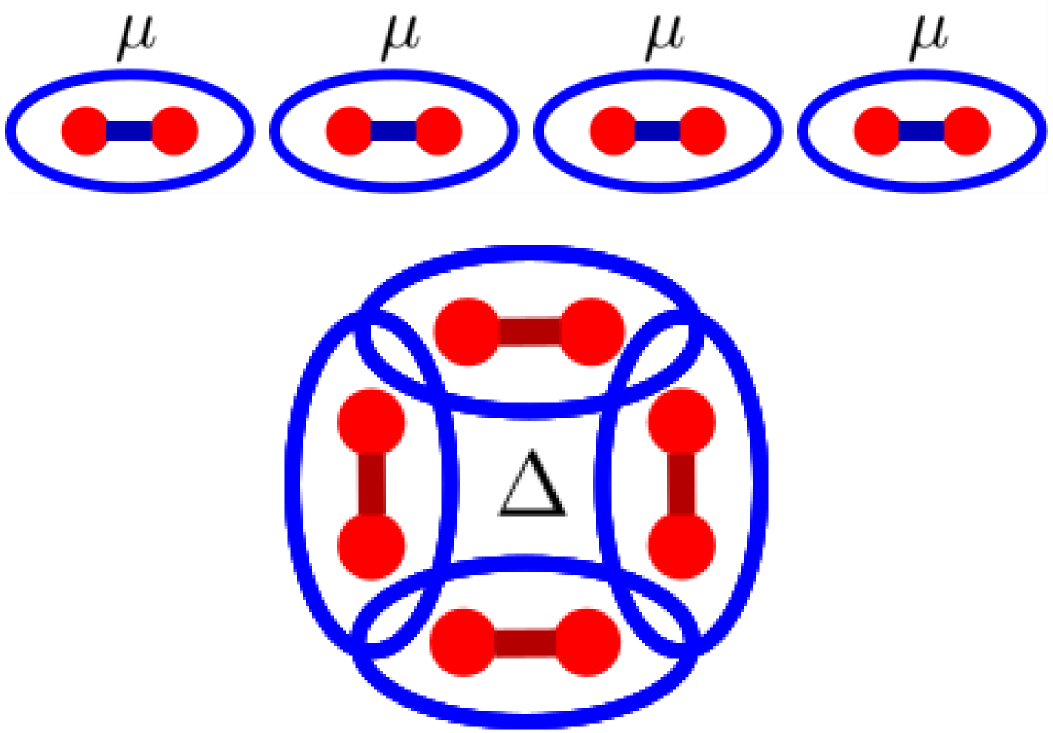
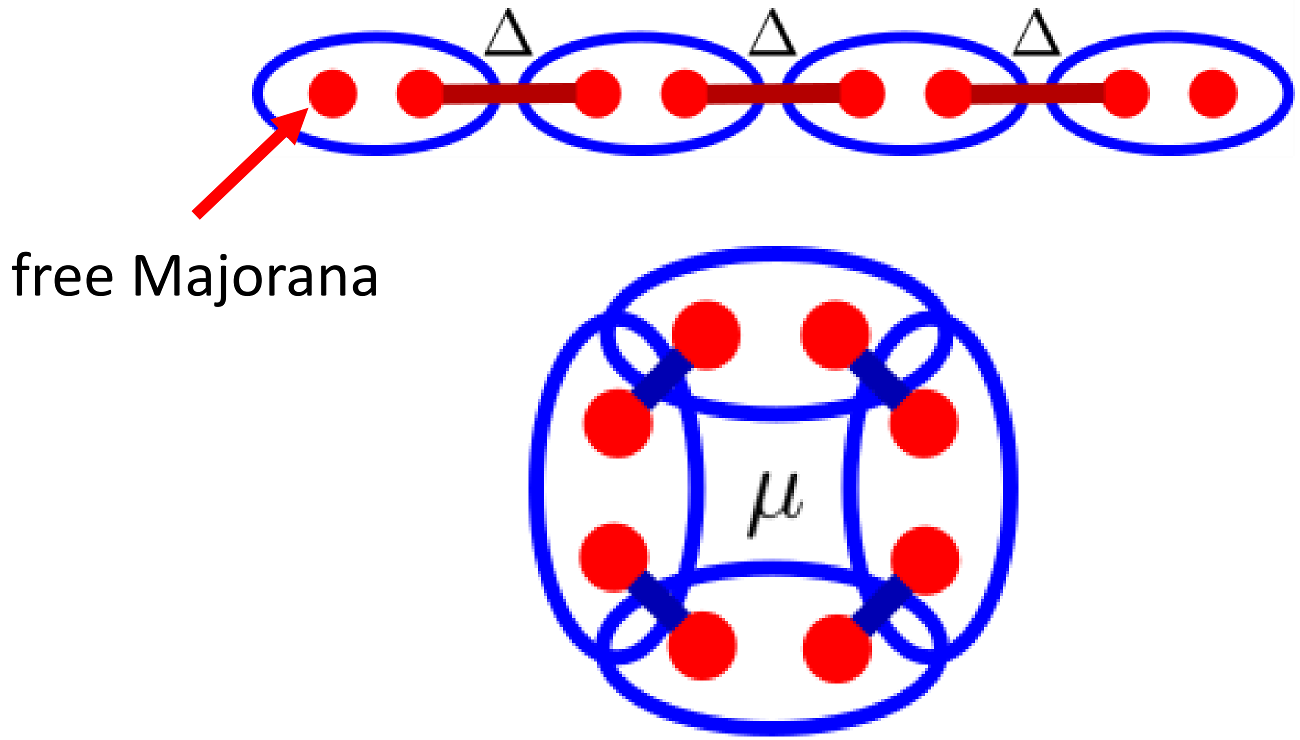
Circular Kitaev chain

fermionic phase $\Delta = 0$



Majorana phase $\mu = 0$

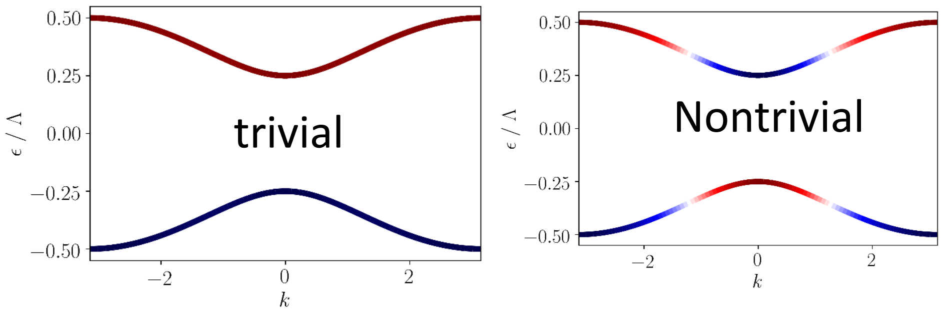


Spectroscopic signatures of topological phases

Unique Signatures of Topological Phases in Two-Dimensional THz Spectroscopy

Felix Gerken, Thore Posske, Shaul Mukamel, and Michael Thorwart

Phys. Rev. Lett. **129**, 017401 – Published 27 June 2022

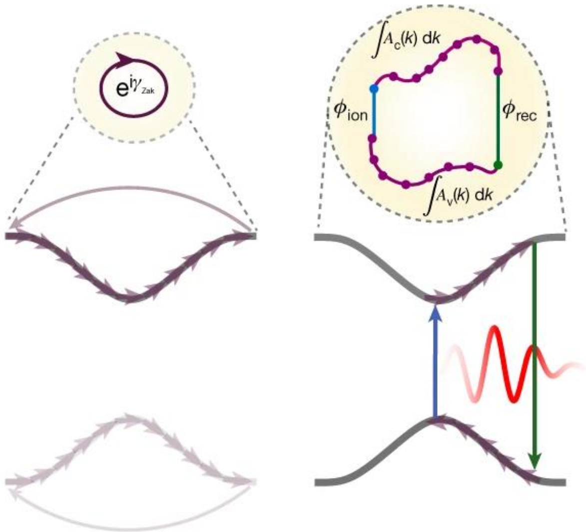


Article | Open access | Published: 17 January 2024

Observation of interband Berry phase in laser-driven crystals

Ayelet J. Uzan-Narovlansky, Lior Faeyrman, Graham G. Brown, Sergei Shames, Vladimir Narovlansky, Jiewen Xiao, Talya Arusi-Parpar, Omer Kneller, Barry D. Bruner, Olga Smirnova, Rui E. F. Silva, Binghai Yan, Álvaro Jiménez-Galán, Misha Ivanov & Nirit Dudovich

Nature **626**, 66–71 (2024) | Cite this article



Kitaev ring (circular Kitaev chain)

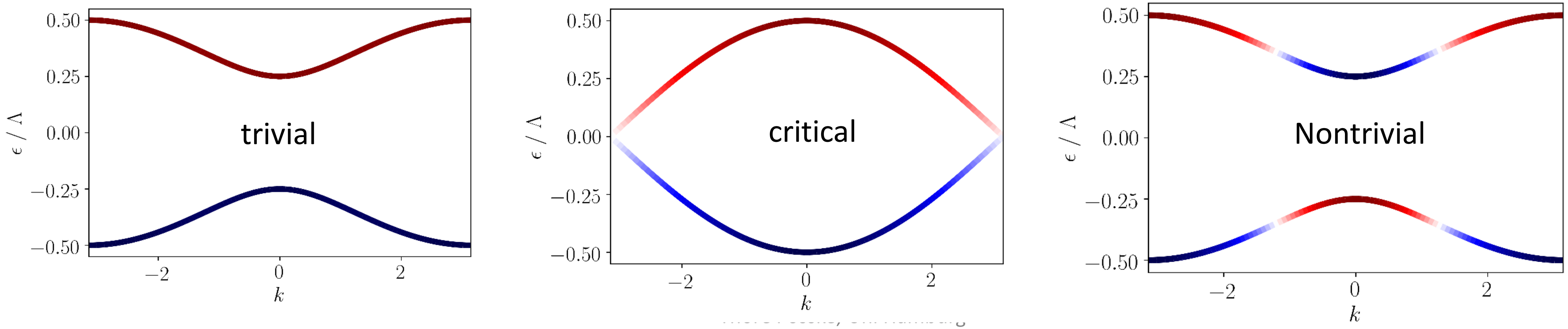
Equivalence transform between topologically trivial and nontrivial phase

$$\mu' = \pm w,\ \ w' = \pm\mu,\ \ \Delta' = e^{i\vartheta}\sqrt{\mu^2 + |\Delta|^2 - w^2}$$

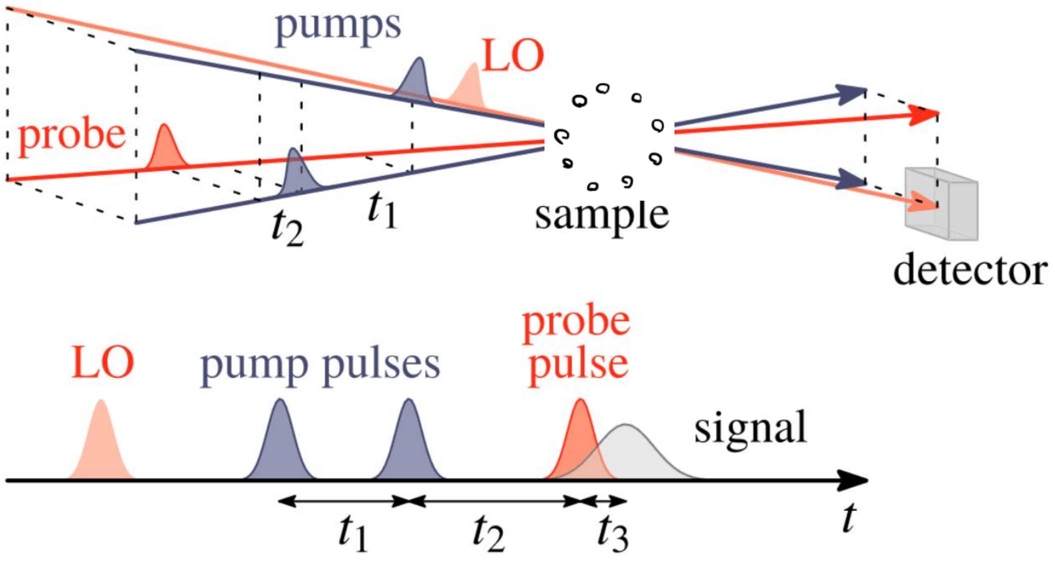
For $|\Delta|^2 > w^2 - \mu^2$

Path with spectrally equivalent Hamiltonians that includes the critical point

$$\Gamma_s = (\mu_s, w_s, \Delta_s) = \Lambda\,(1 - s,\ s,\ s)\,/2$$



Multi-pulse Spectroscopy



Gelzini et al. BBA-Bioenergetics **1860**, 271 (2019)

Time-dependent electric-field

$$V(t) = -\mathbf{d} \cdot \mathbf{E}(t) \qquad \mathbf{d} = -e\mathbf{R}$$

Measure polarization

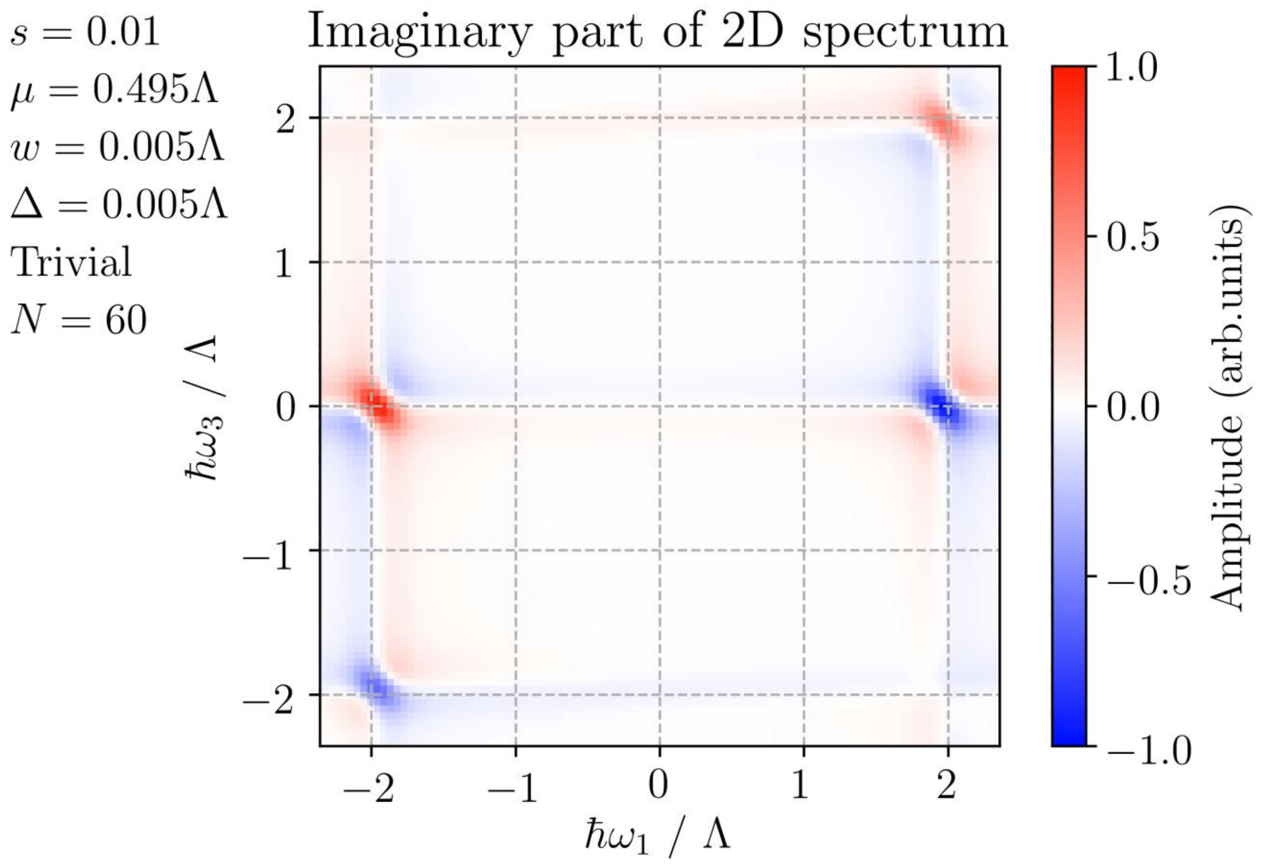
$$P^{(3),j}(t) = \int_0^\infty dt_3 dt_2 dt_1 E^m(t-t_3) E^l(t-t_3-t_2) \times E^k(t-t_3-t_2-t_1) S^{(3),j}_{klm}(t_3,t_2,t_1)$$

Obtain "2D spectrum"

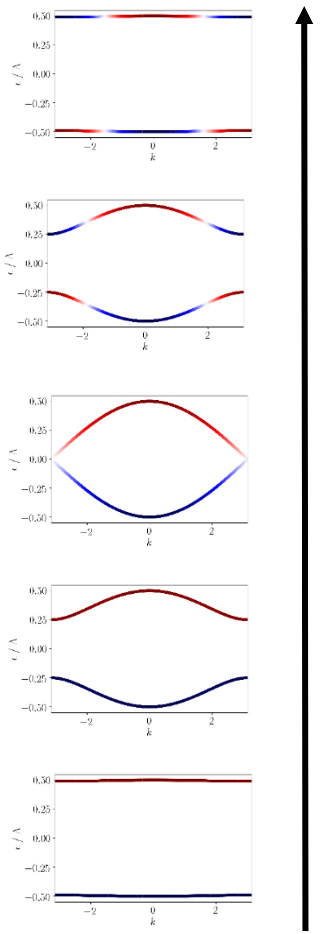
$$\mathrm{Im}\ S^{(3),j}_{klm}(\omega_3, t_2, \omega_1)$$

We set $t_2 = 0$.

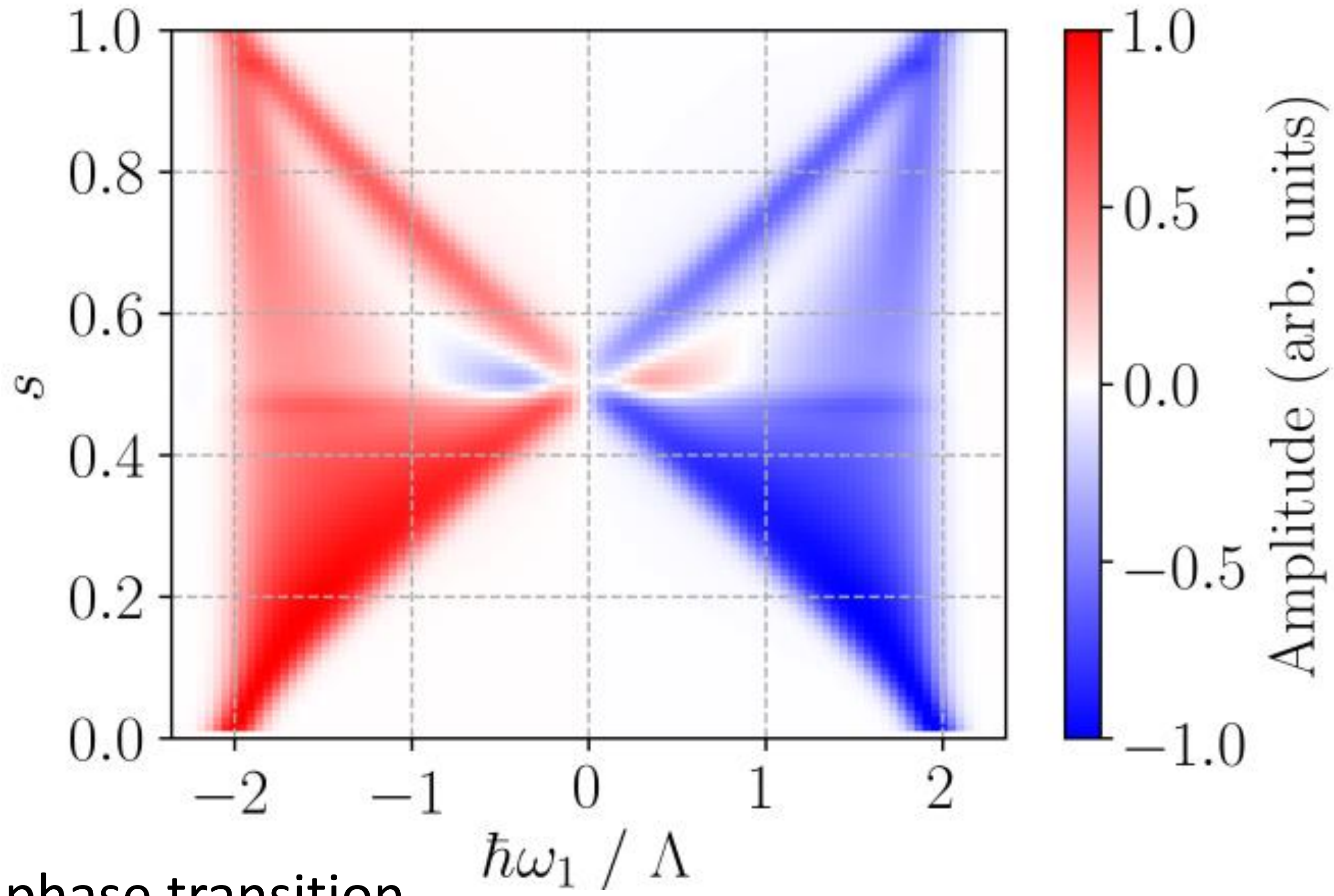
Multi-pulse spectroscopy of a topological phase transition



Sections of the 2D spectrum



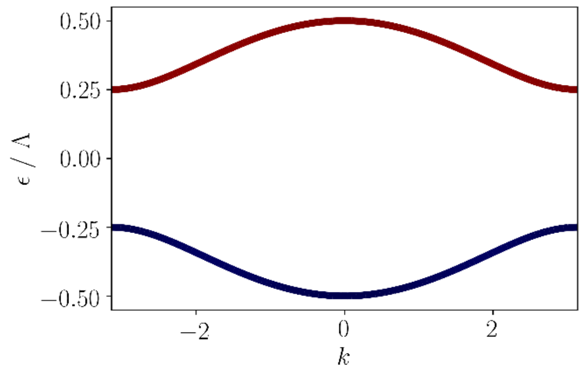
Horizontal Section



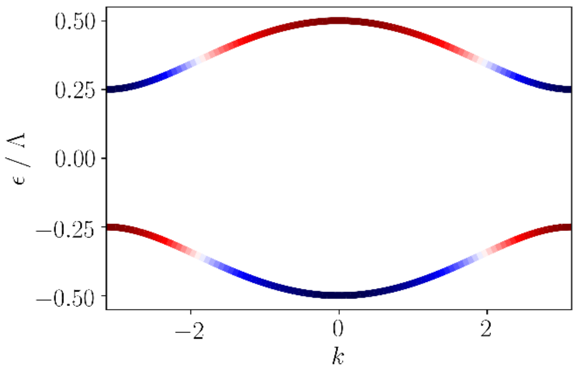
Tune through phase transition

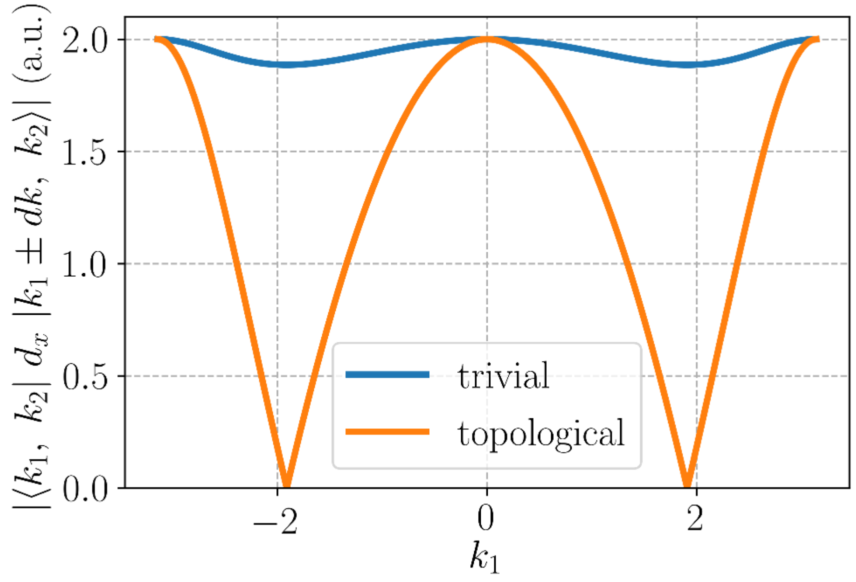
Detecting topological band inversion

Trivial Phase



Topological Phase





2D Spectroscopy detects Particle-hole inversion

Away from phase transition

2-particle to 2-particle transitions:

$$\langle k_1, k_2|\, d_x \,|k_1 \pm dk, k_2\rangle$$

Closure of dipole gap

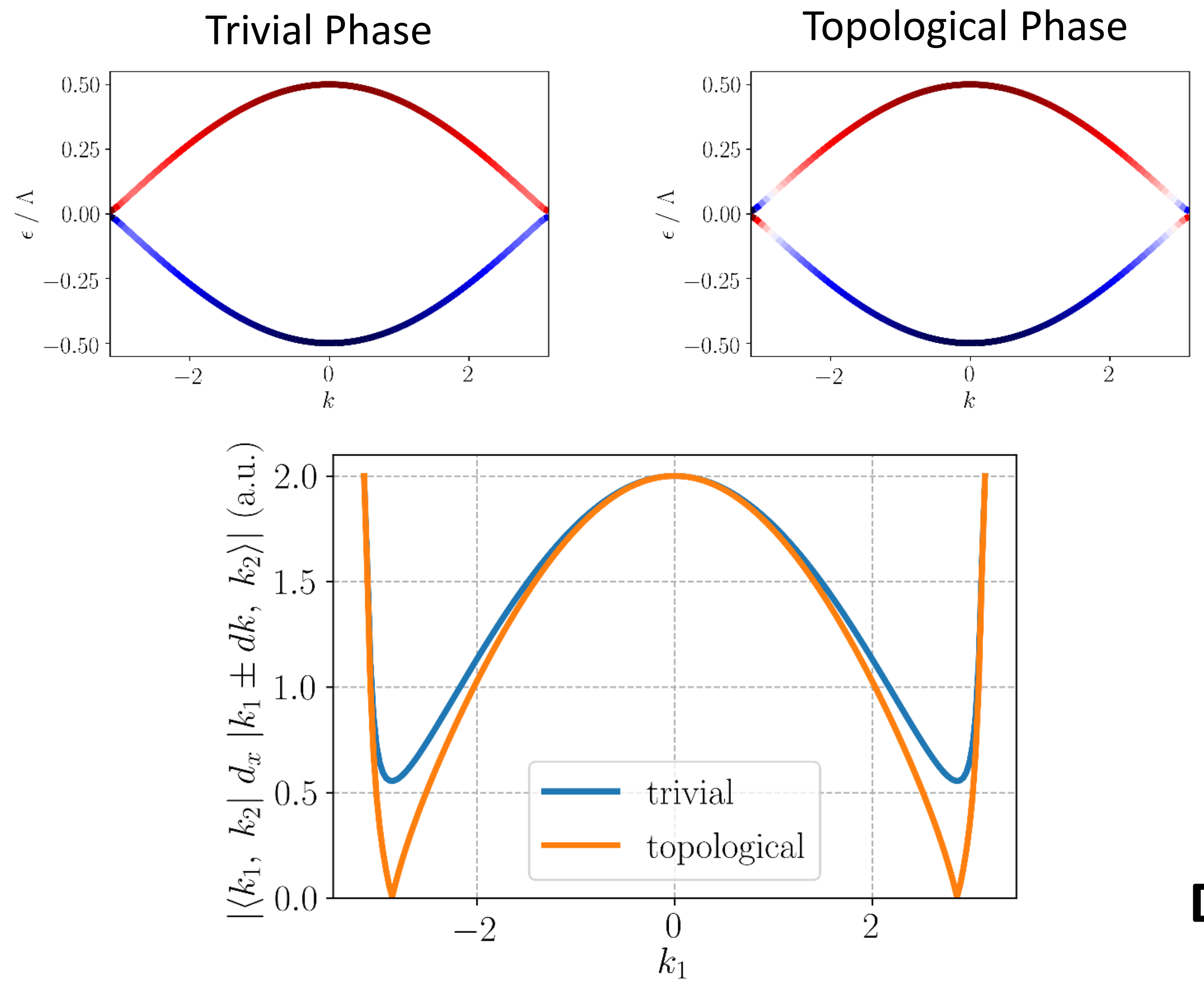
⇔

topological band inversion

⇔

Double peak on horizontal in 2D spectrum

Detecting topological band inversion



2D Spectroscopy detects Particle-hole inversion

Close to phase transition

2-particle to 2-particle transitions:

$$\langle k_1, k_2 |\, d_x \,| k_1 \pm dk, k_2 \rangle$$

Closure of dipole gap

⇔

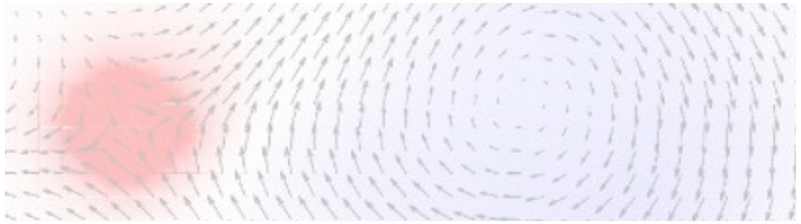
topological band inversion

⇔

Double peak on horizontal in 2D spectrum

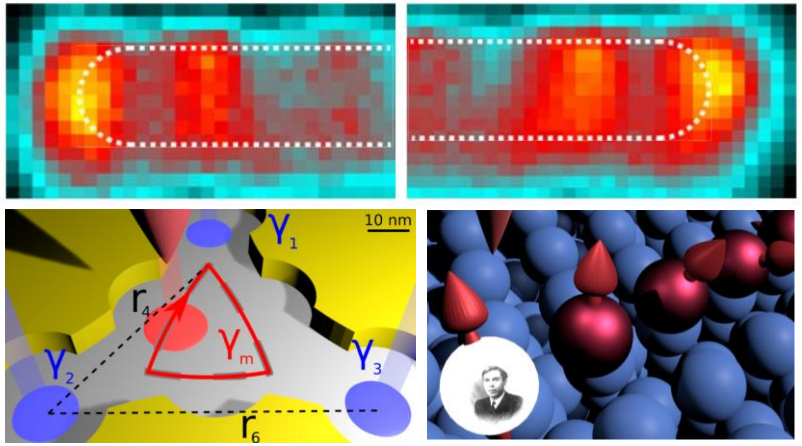
Recent research

Topological magnetism



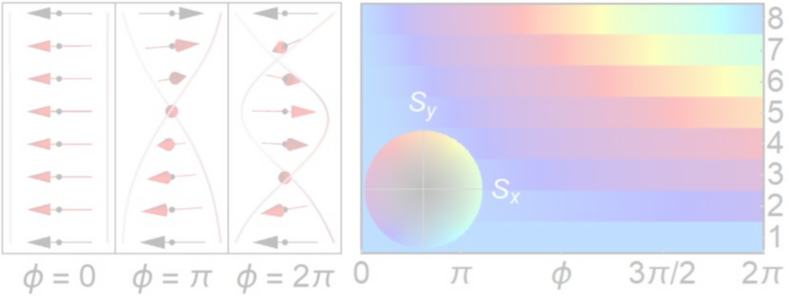
PRL (2017)

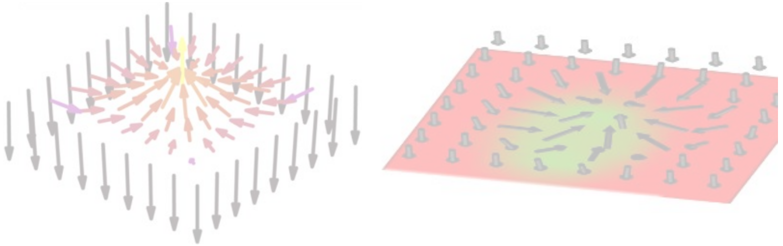
Topological phases & Majoranas



Science Adv. (2018), **Nat. Phys.** (2021), **Nat. Nano.** (2022), **Nature** (2023), **US patent** (2024)

Topological quantum magnetism





PRL (2019), **DFG** (2020), **ERC** (2023)

Anyons

$$a_p^\dagger a_q^\dagger = e^{i\phi_\eta(p-q)} a_q^\dagger a_p^\dagger,$$

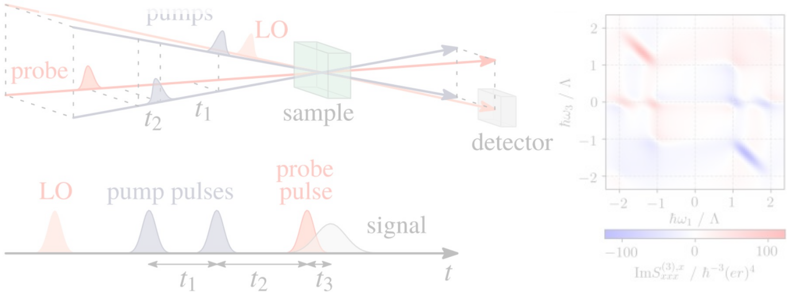
$$a_p a_q^\dagger = e^{-i\phi_\eta(p-q)} a_q^\dagger a_p + \delta(p-q)$$

PRL (2021)

Kondo effect

PRL (2013), **PRL** (2015)

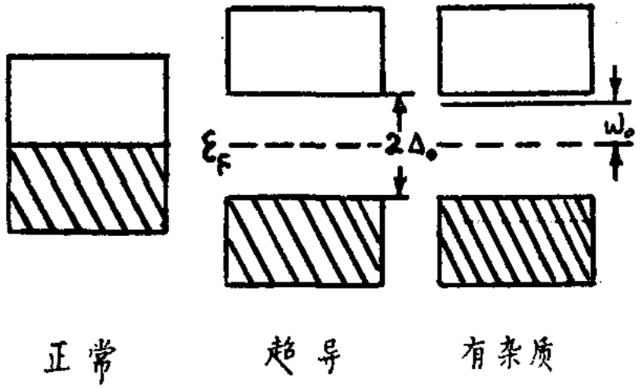
Spectroscopy + top. matter



PRL (2022)

Superconductor surfaces as platform for quantum states

Magnetic adatoms induce **Yu-Shiba-Rusinov states**



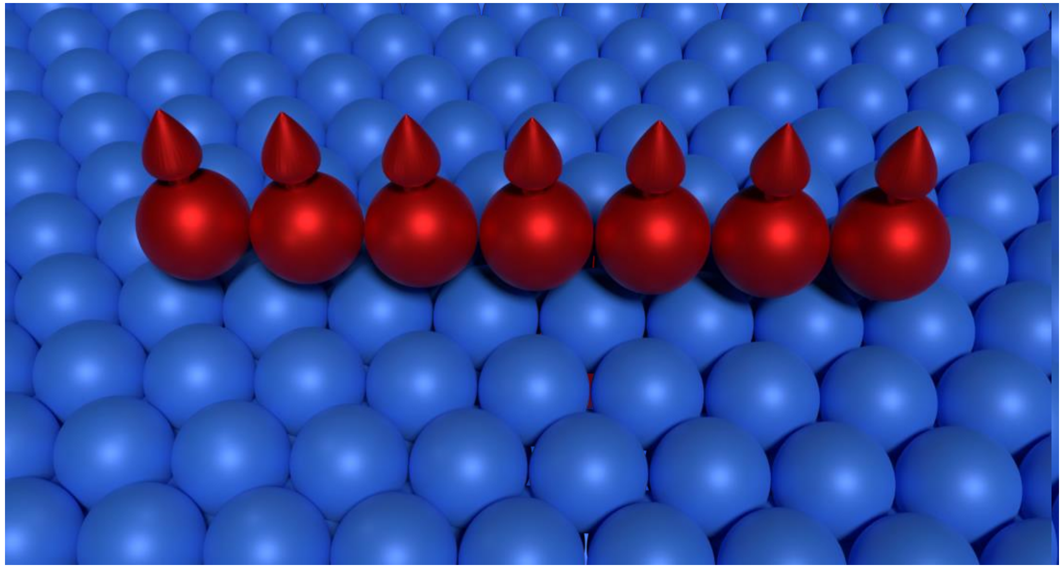
L. **Yu**, Acta Phys. Sin. 21, 75 (1965)
H. **Shiba**, Prog. Theor. Phys. 40, 435 (1968)
A. I. **Rusinov**, ZhETF Pis. Red. 9, 146 (1969)
[JETP Lett. 9, 85 (1969)]

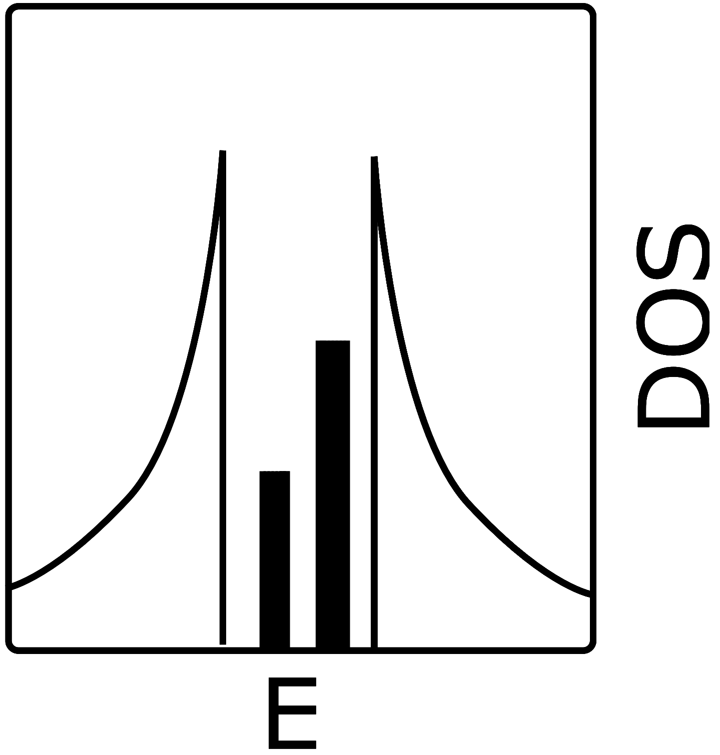
$$H_{\rm int} = \int V(\mathbf{r})[\psi_\uparrow^+(\mathbf{r})\psi_\uparrow(\mathbf{r}) + \psi_\downarrow^+(\mathbf{r})\psi_\downarrow(\mathbf{r})]d^3\mathbf{r} - $$
$$- \int J(\mathbf{r})\{\hat{S}_x[\psi_\uparrow^+(\mathbf{r})\psi_\uparrow(\mathbf{r}) - \psi_\downarrow^+(\mathbf{r})\psi_\downarrow(\mathbf{r})] + \hat{S}_+\psi_\downarrow^+(\mathbf{r})\psi_\uparrow(\mathbf{r}) +$$
$$+ \hat{S}_-\psi_\uparrow^+(\mathbf{r})\psi_\downarrow(\mathbf{r})\}d^3\mathbf{r}, \qquad (3)$$

这里 $V(\mathbf{r})$ 是散射势；$J(\mathbf{r})$ 是交换积分，$\hat{S}_x$ 和 $\hat{S}_\pm = \hat{S}_x \pm i\hat{S}_y$ 是杂質原子的自旋分量算符.

Superconductor surfaces as platform for quantum states

Magnetic adatoms induce Yu-Shiba-Rusinov states
Superconducting gap shields from electronic noise
Atomically precise control by scanning tunneling microscopy



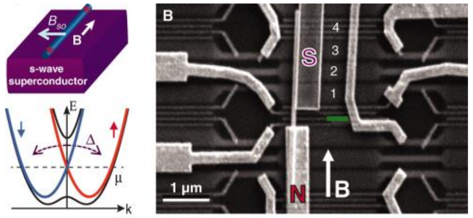


Low-dimensional topological superconductors

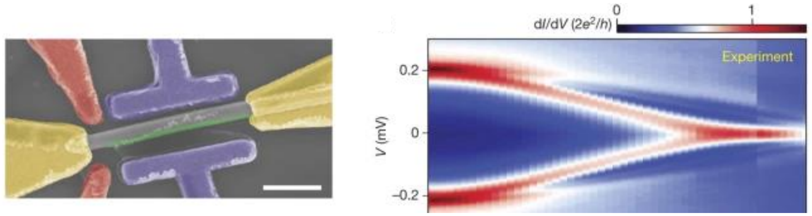
Symmetry				Dimension							
AZ	T	C	S	1	2	3	4	5	6	7	8
A	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AIII	0	0	1	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
AI	1	0	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
BDI	1	1	1	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
D	0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$
DIII	-1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0
AII	-1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$
CII	-1	-1	1	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
C	0	-1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
CI	1	-1	1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0

Problems with disorder
@spinespresso
Sergey Frolov

Mourik et al. Science 2012



Zhang et al. Nature (2018)



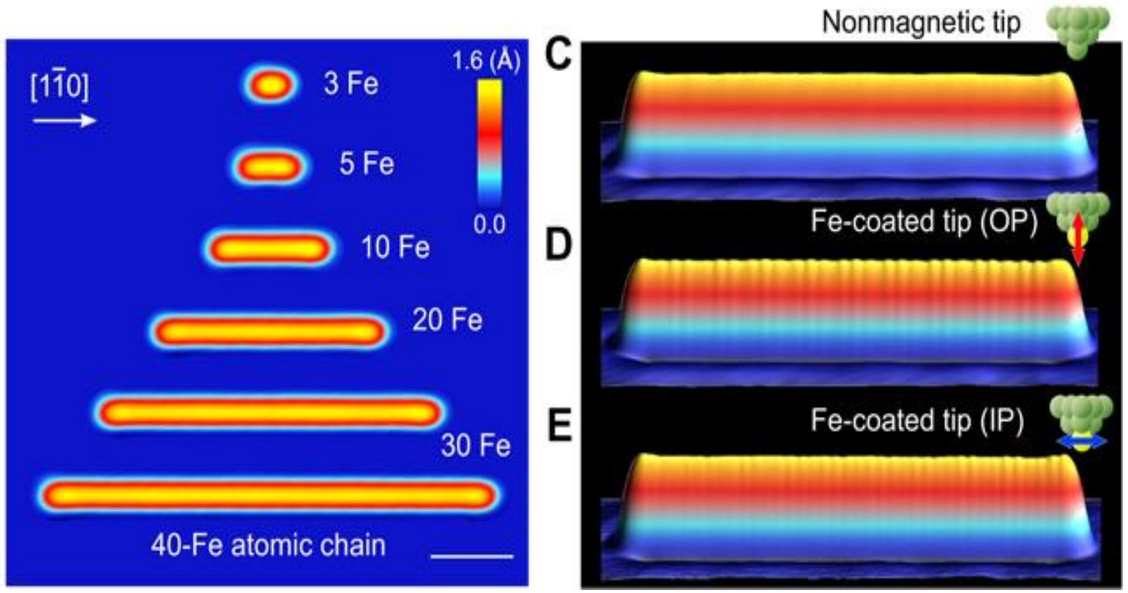
Retraction Note | Published: 08 March 2021

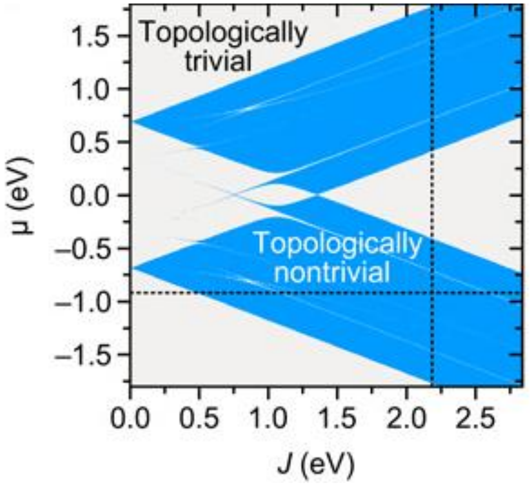
Retraction Note: Quantized Majorana conductance

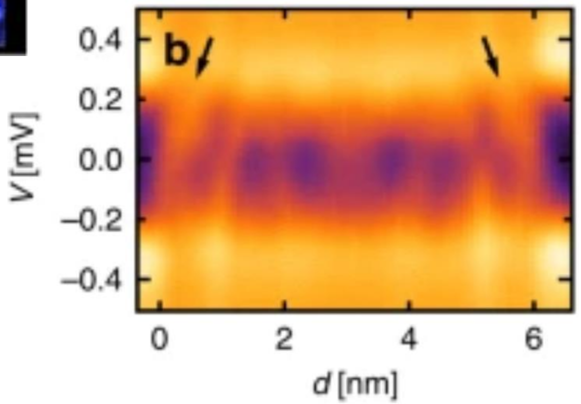
Hao Zhang, Chun-Xiao Liu, Sasa Gazibegovic, Di Xu, John A. Logan, Guanzhong Wang, Nick van Loo, Jouri D. S. Bommer, Michiel W. A. de Moor, Diana Car, Roy L. M. Op het Veld, Petrus J. van Veldhoven, Sebastian Koelling, Marcel A. Verheijen, Mihir Pendharkar, Daniel J. Pennachio, Borzoyeh Shojaei, Joon Sue Lee, Chris J. Palmstrøm, Erik P. A. M. Bakkers, S. Das Sarma & Leo P. Kouwenhoven

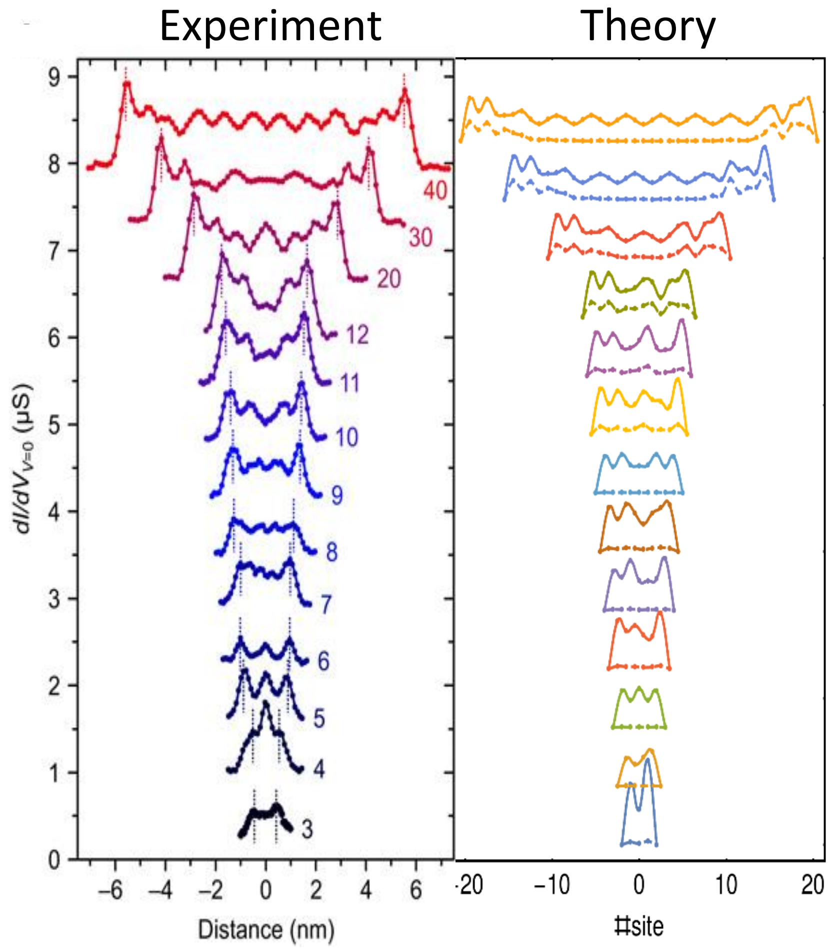
Nature **591**, E30 (2021) | Cite this article

Towards 1D topological SC in adatom systems – Fe on Re







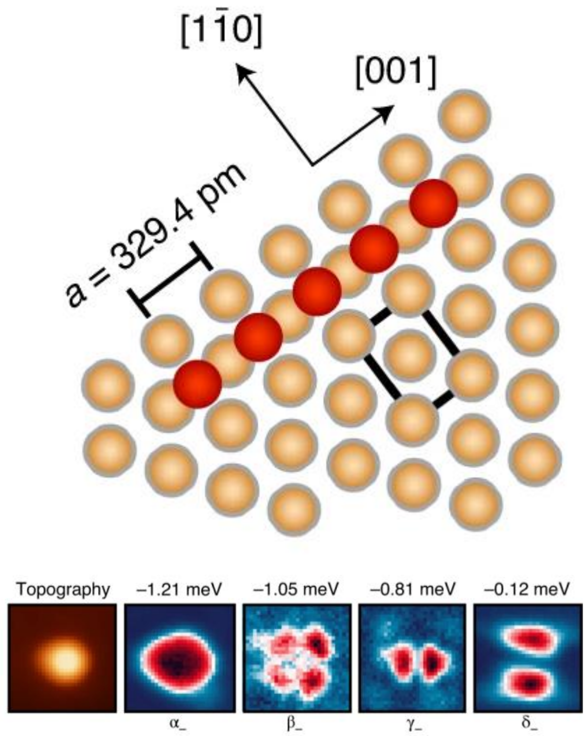


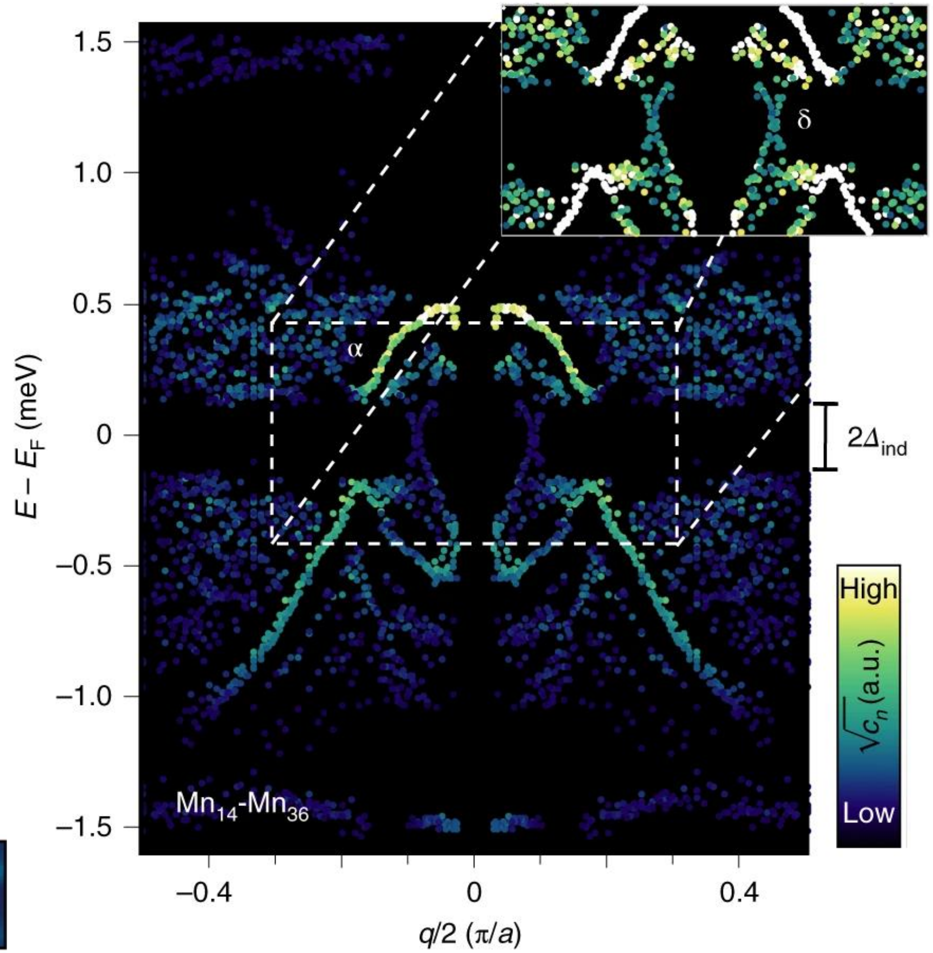
- No disorder
- Spin spiral in Fe chain on Re
- Tight-binding model with first principle parameters
- No hard gap

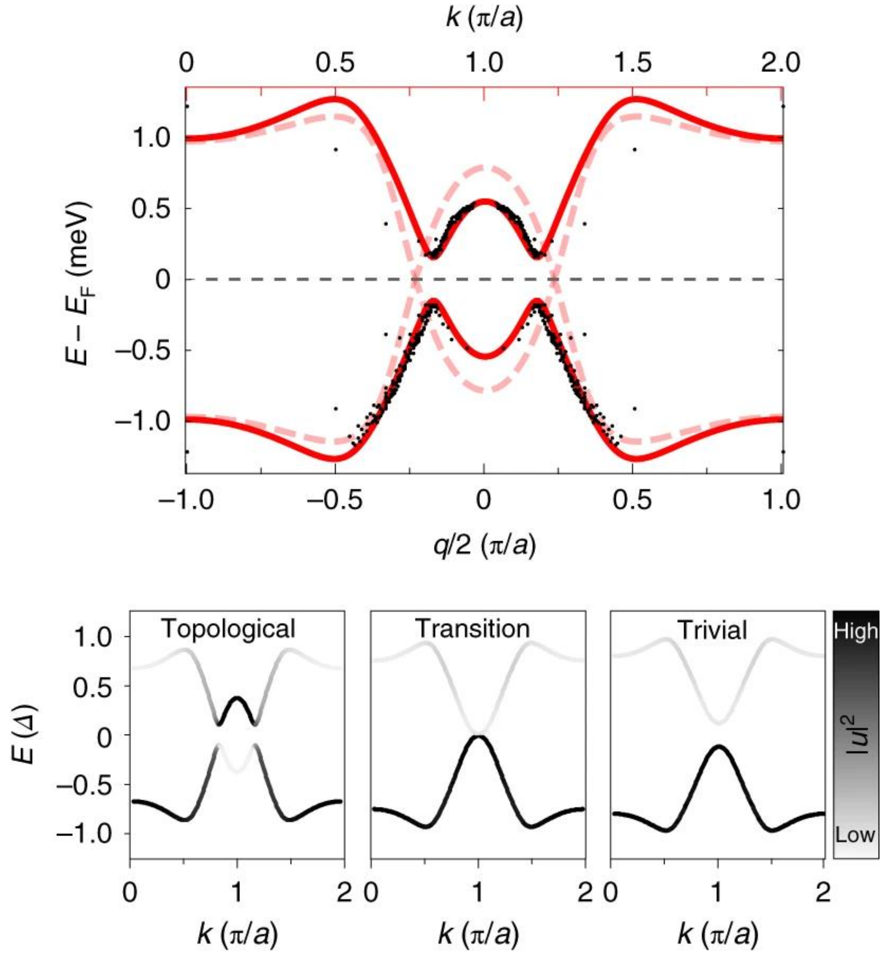
Kim, …, Posske, ..., Thorwart, …, Wiesendanger, Science Adv. (2018)

Towards 1D topological SC in adatom systems – Mn[001] on Nb(110)

Mn[001]



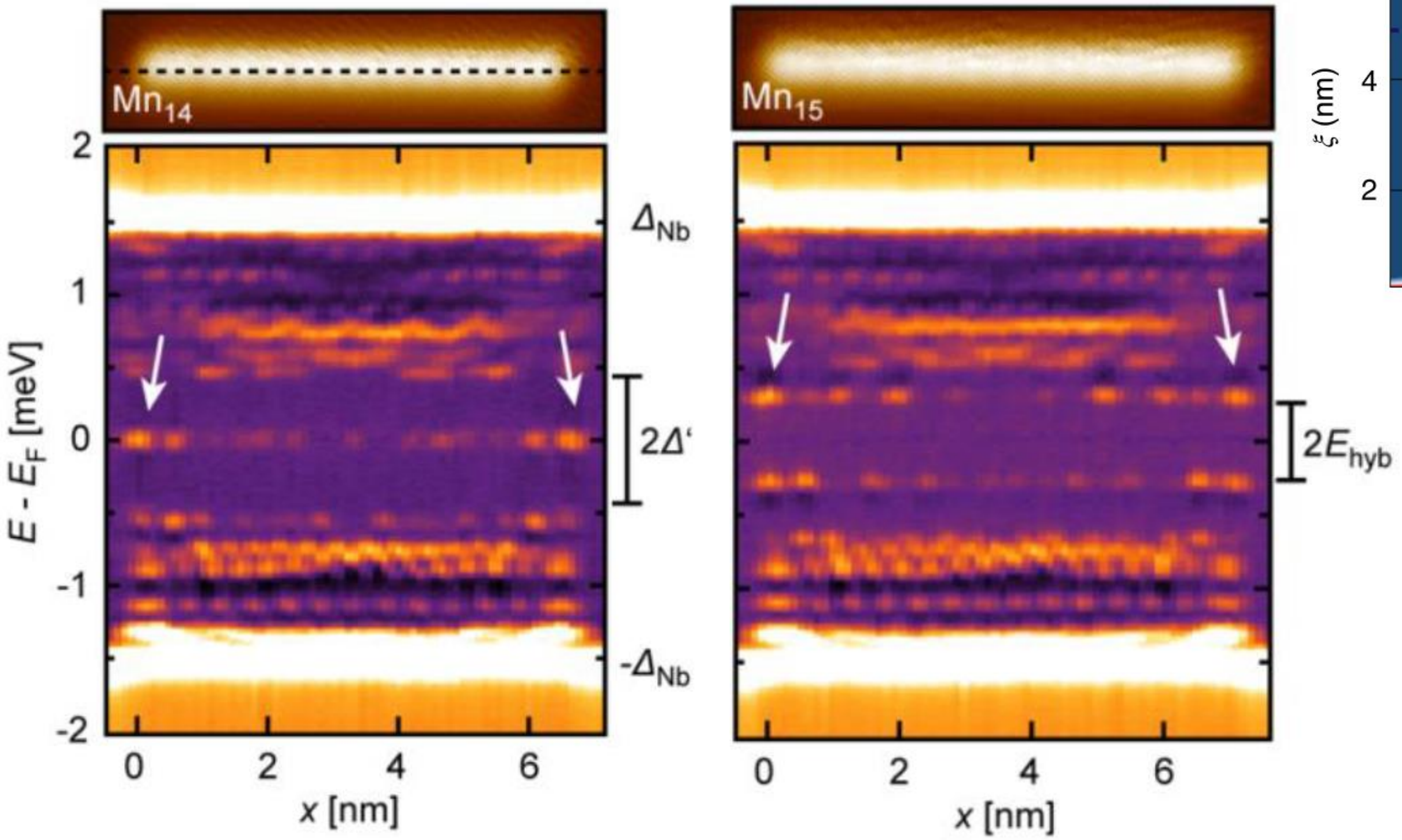


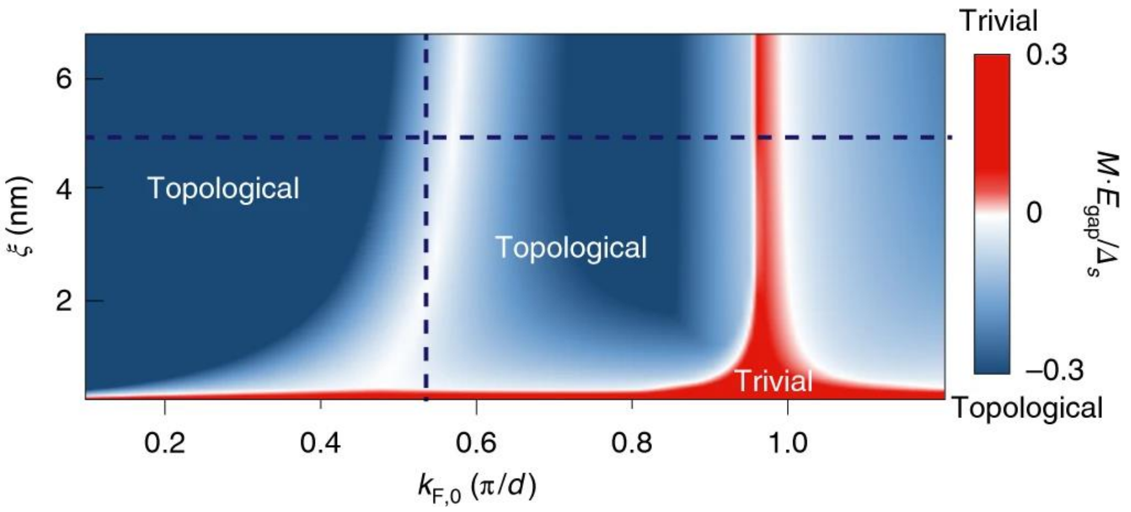


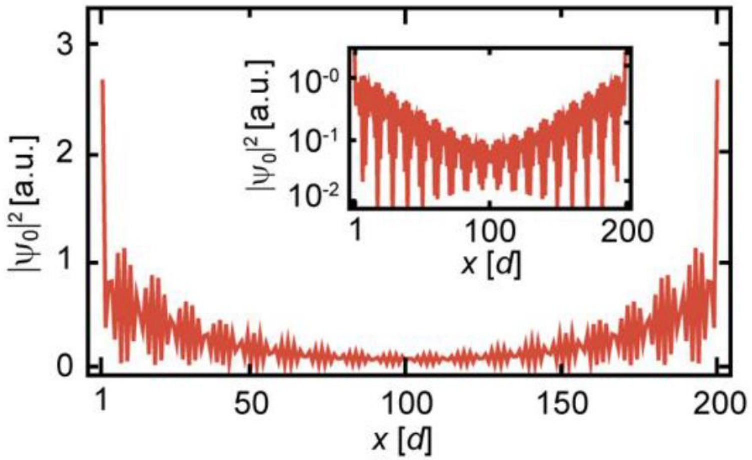
Schneider, .., Posske, .., Wiesendanger Nat. Phys. **17,** 943 (2021)

Towards 1D topological SC – Mn[1 -1 0] on Nb(110)

Precursors of Majorana modes



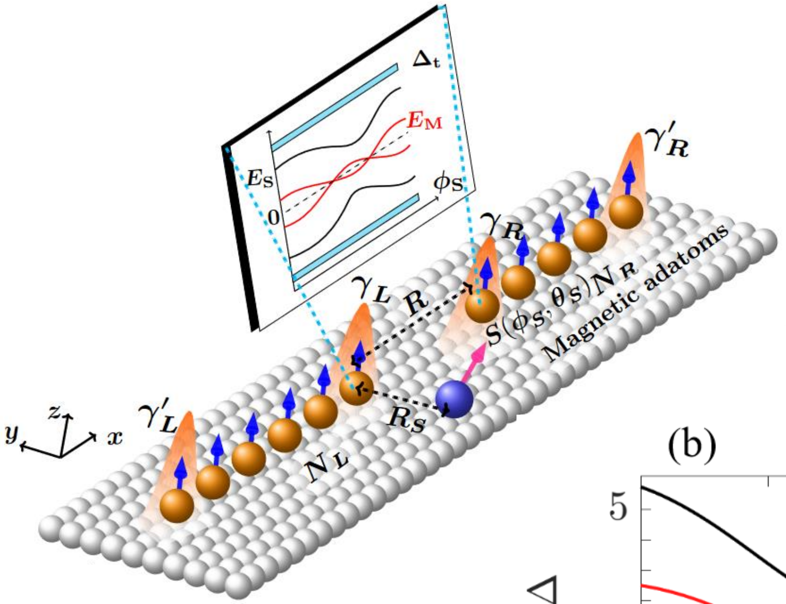


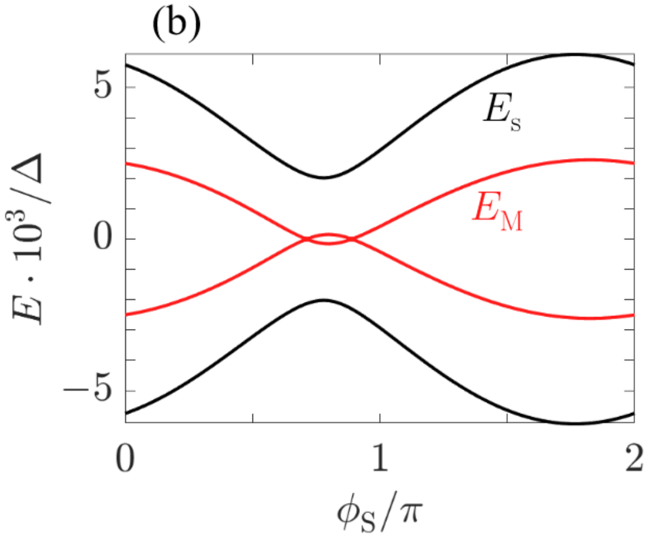


Schneider, .., Posske, .., Wiesendanger Nat. Nano. **17,** 384 (2022)

Selected Majorana projects

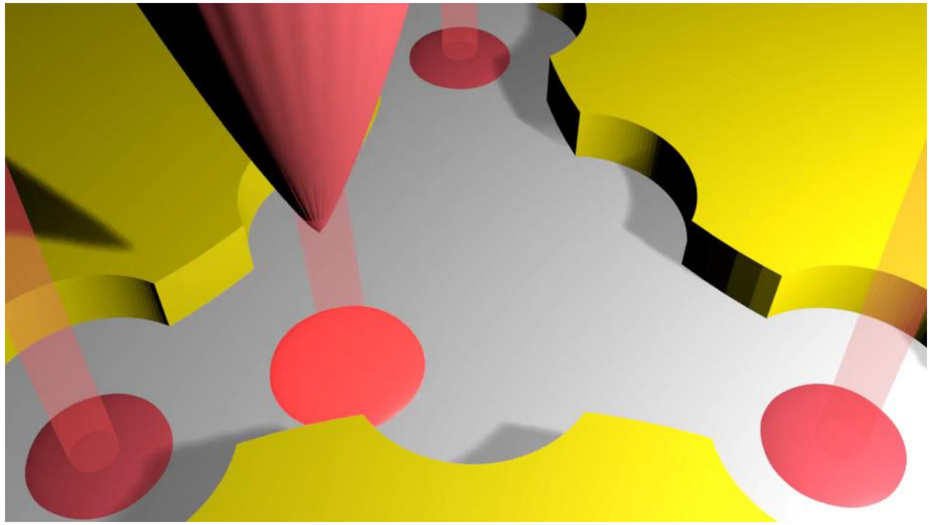
Majorana shielding



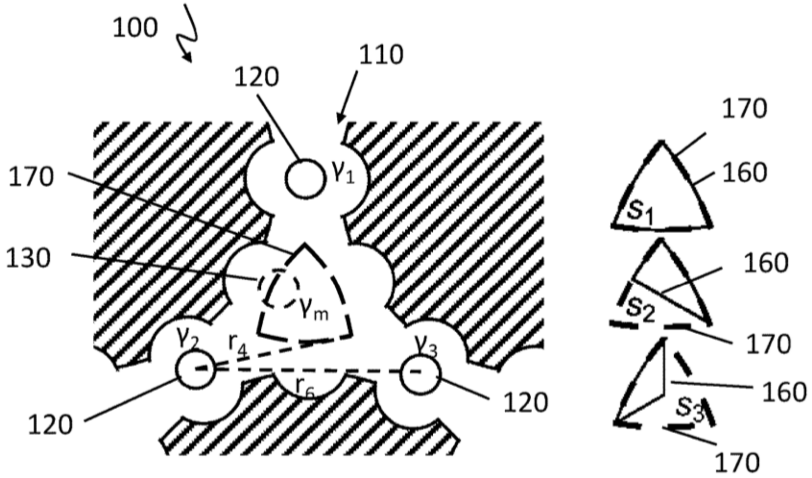


Awoga, .., Posske, arxiv (2023)

High-frequency Majorana braiding

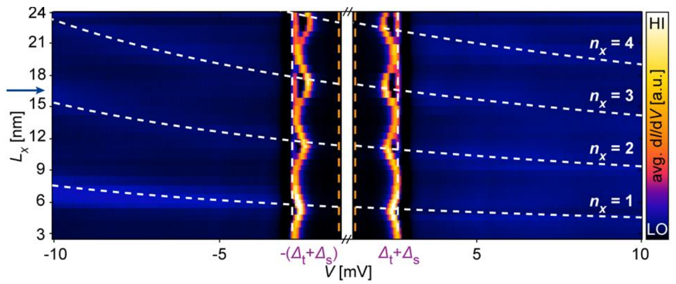


Posske, Chiu, Thorwart, PRR **2**, 023205 (2020)

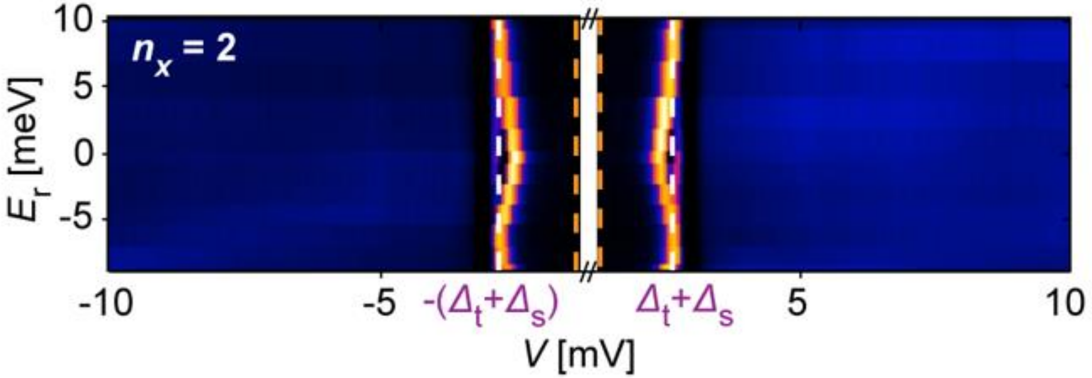


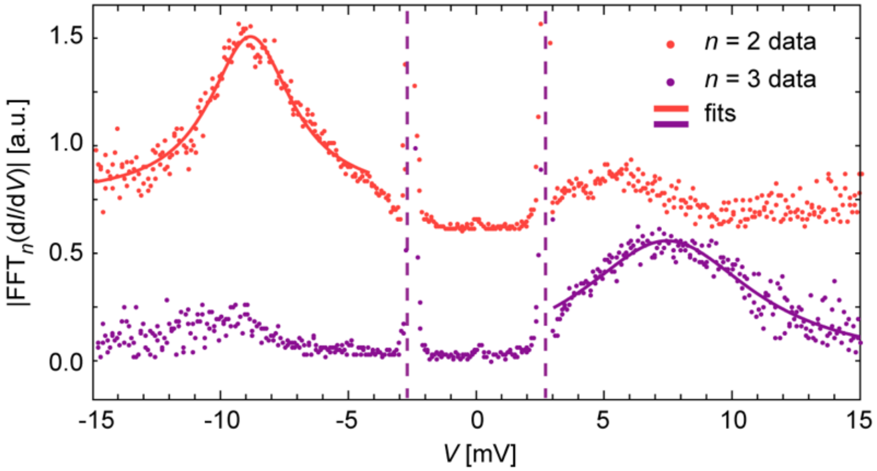
US patent (2024)

Measurements

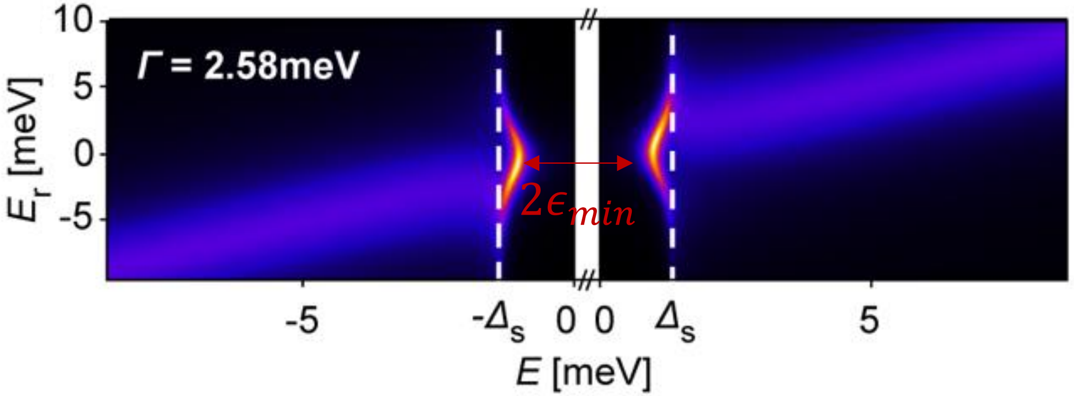


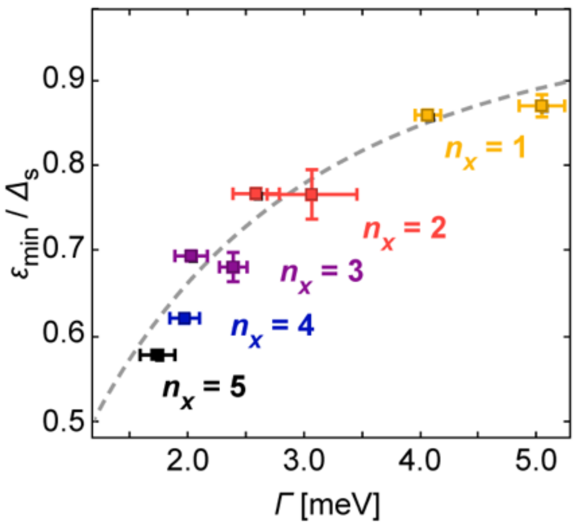
Experiment

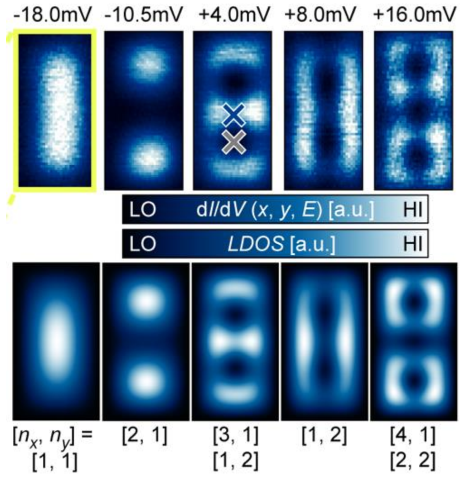




Theory







Single level superconductivity details

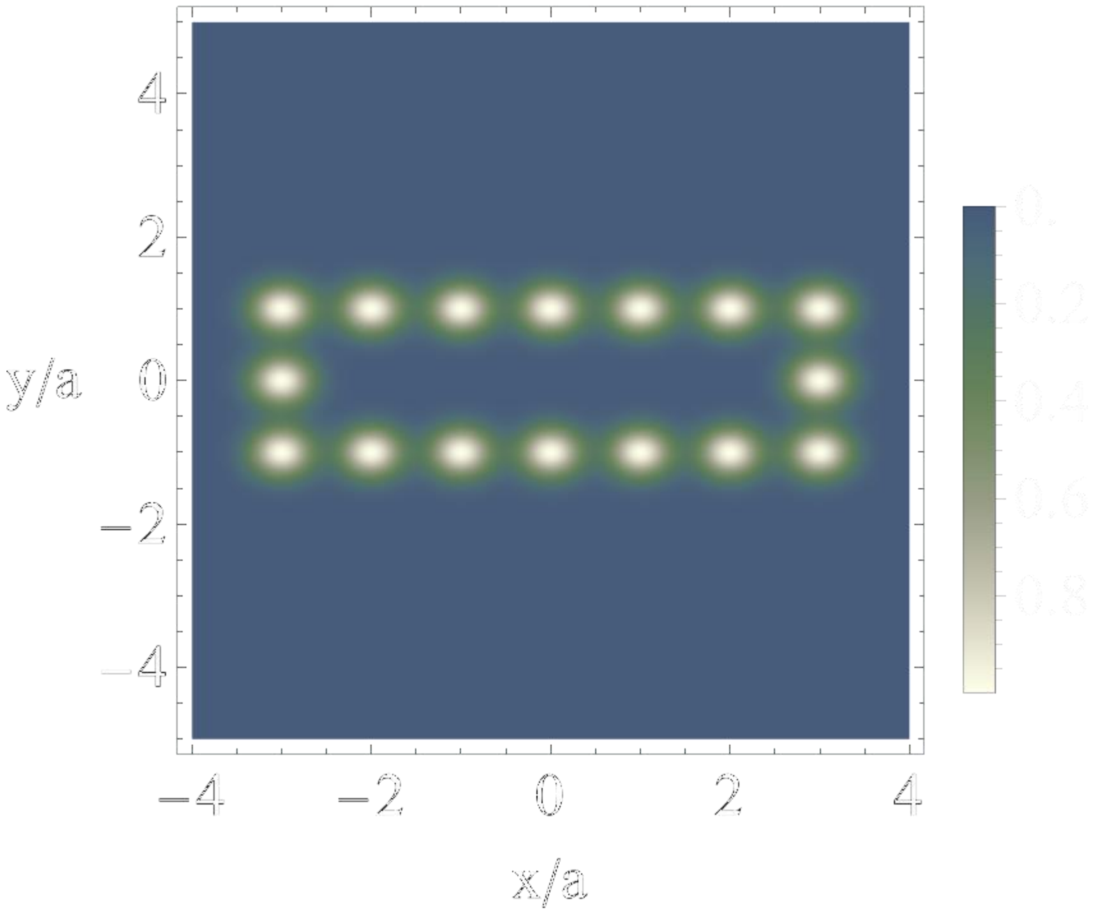
MSS energies: $\varepsilon = \pm\sqrt{{E_\mathrm{r}}^2(1 - \Delta_\mathrm{ind}/\Delta_\mathrm{s})^2 + {\Delta_\mathrm{ind}}^2}$

Particle-hole weight: $|v|^2 = \frac{1}{2} - \frac{E_\mathrm{r}\left(1 - \frac{\Delta_\mathrm{ind}}{\Delta_\mathrm{s}}\right)}{2\varepsilon}$ $\quad |u|^2 = 1 - |v|^2$

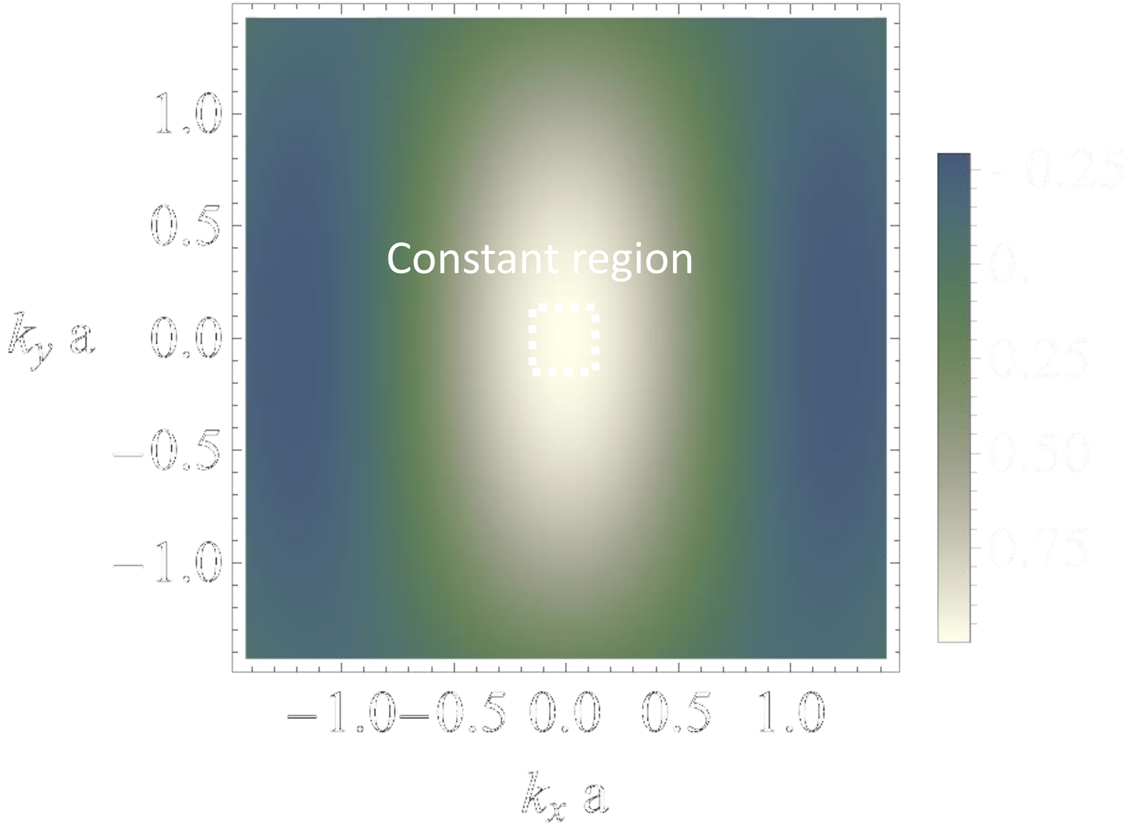
Bogoliubov mixing angle: $\theta_\mathrm{B}(\varepsilon) = \mathrm{ArcTan}\left(\sqrt{|u|^2/|v|^2}\right)$

Fourier transform of potential

Real space

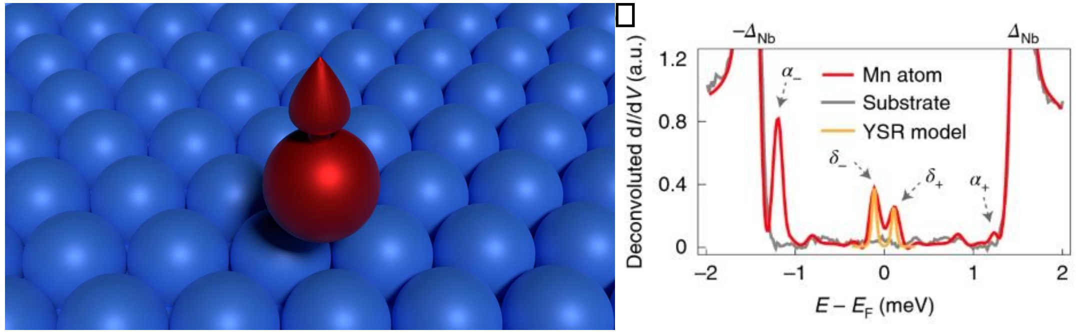


Fourier space



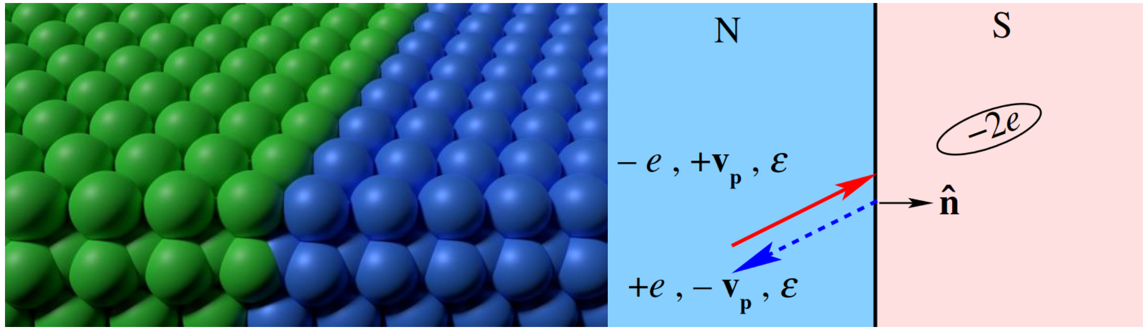
In gap states in superconductors

Yu-Shiba Rusinov states



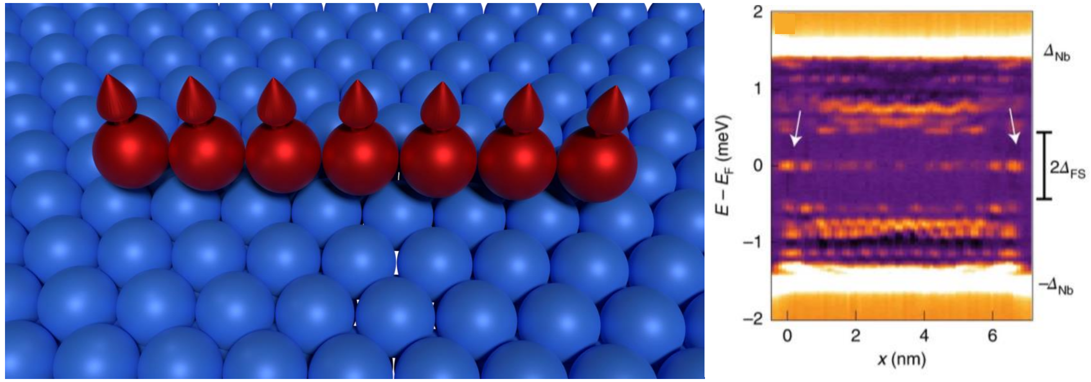
Schneider et al., *Nat. Nano* **17**, 384 (2022)

Andreev bound states



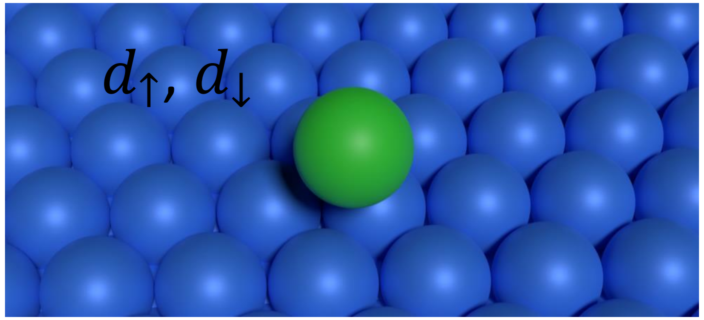
Sauls, Phil. Trans.R. Soc. **A376**, 20180140 (2018)

Majoranas and their precursors



Schneider et al., *Nat. Nano* **17**, 384 (2022)

Machida-Shibata states

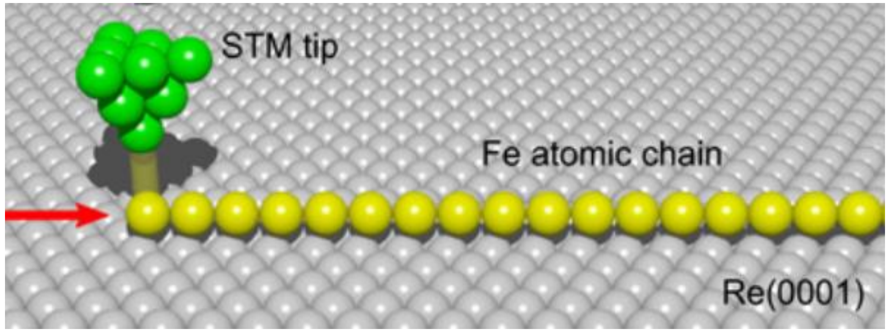


Schneider et al. arXiv:2212.00657 (2022)

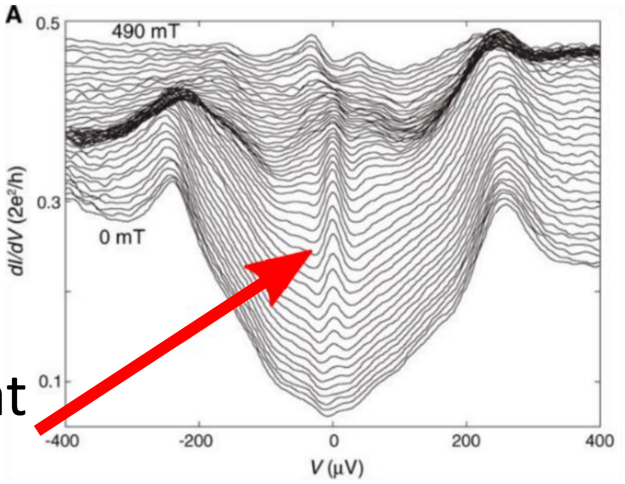
Majorana zero modes

- Zero energy electronic mode found in
  - Semiconductor wires and atomic chains on superconductors + magnetic field
  - Superconducting vortices
- Proposals for driven and not driven cold atoms

Designed atomic chains



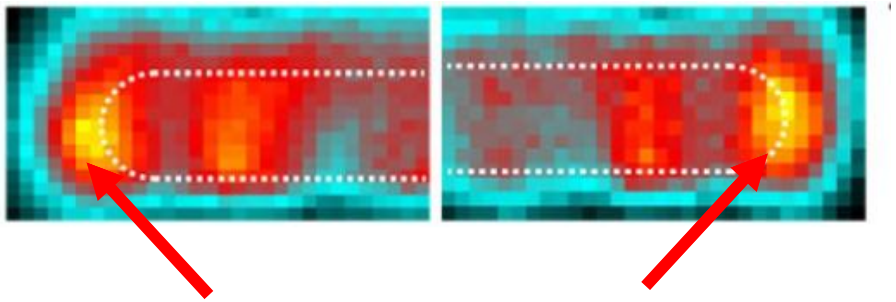
Kim, ..., Posske, ..., Thorwart, ..., Wiesendanger, Science Adv. (2018)



Gapped peak at zero energy

Mourik et al. (Kouwenhoven), *Science* (2012)

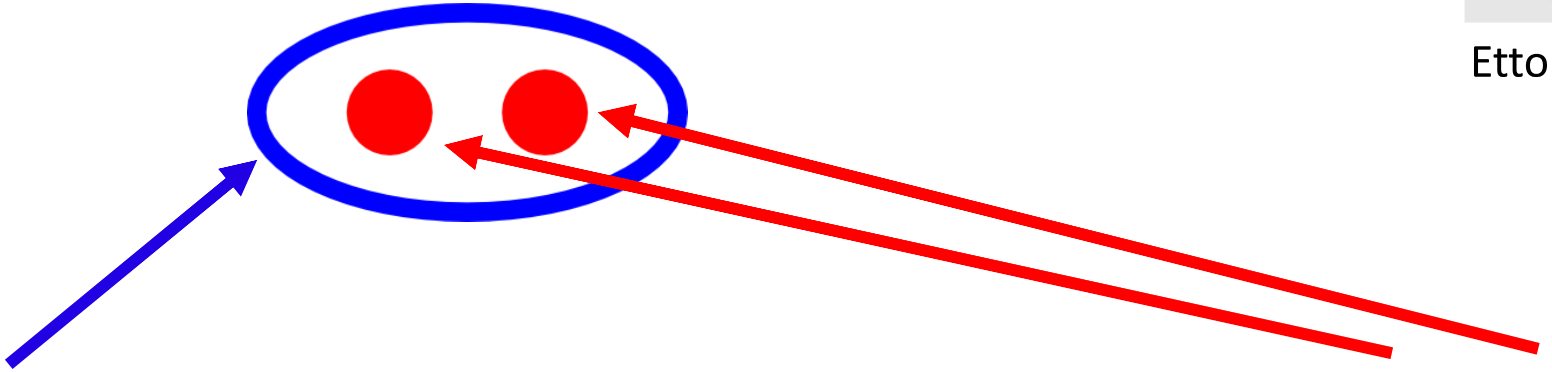
Scanning tunneling microscopy



Majorana mode at edge

Majorana – half a fermion

Ettore Majorana



fermion operator c

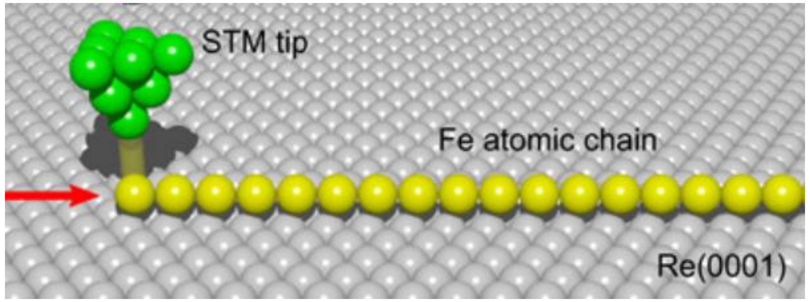
Majorana operator γ_A, γ_B

$$\{c, c^\dagger\} = 1$$
$$\{c, c\} = 0$$

$$\gamma_A = \frac{1}{\sqrt{2}}\left(c + c^\dagger\right)$$
$$\gamma_B = \frac{1}{\sqrt{2}\mathrm{i}}\left(c - c^\dagger\right)$$

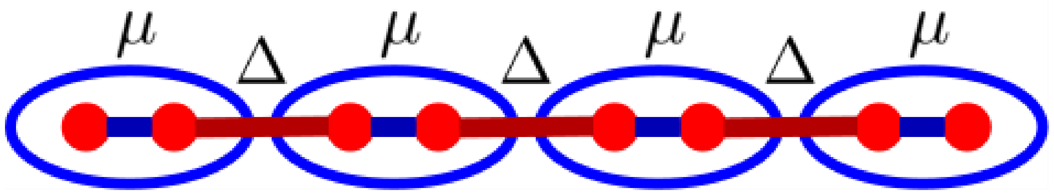
$$\left\{\gamma_i, \gamma_j^\dagger\right\} = \delta_{i,j}$$
$$\gamma_j^\dagger = \gamma_j$$

1D superconductors



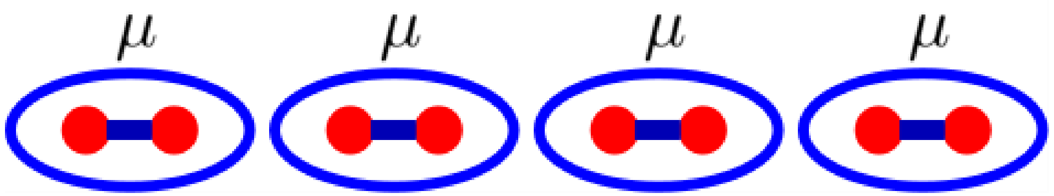
μ chemical potential

Δ superconductivity and hopping



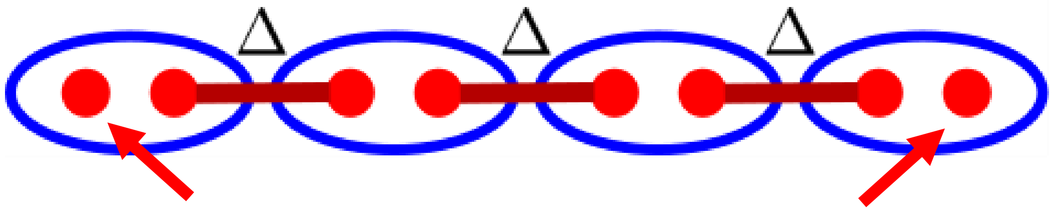
fermionic phase

$\Delta = 0$



Majorana phase

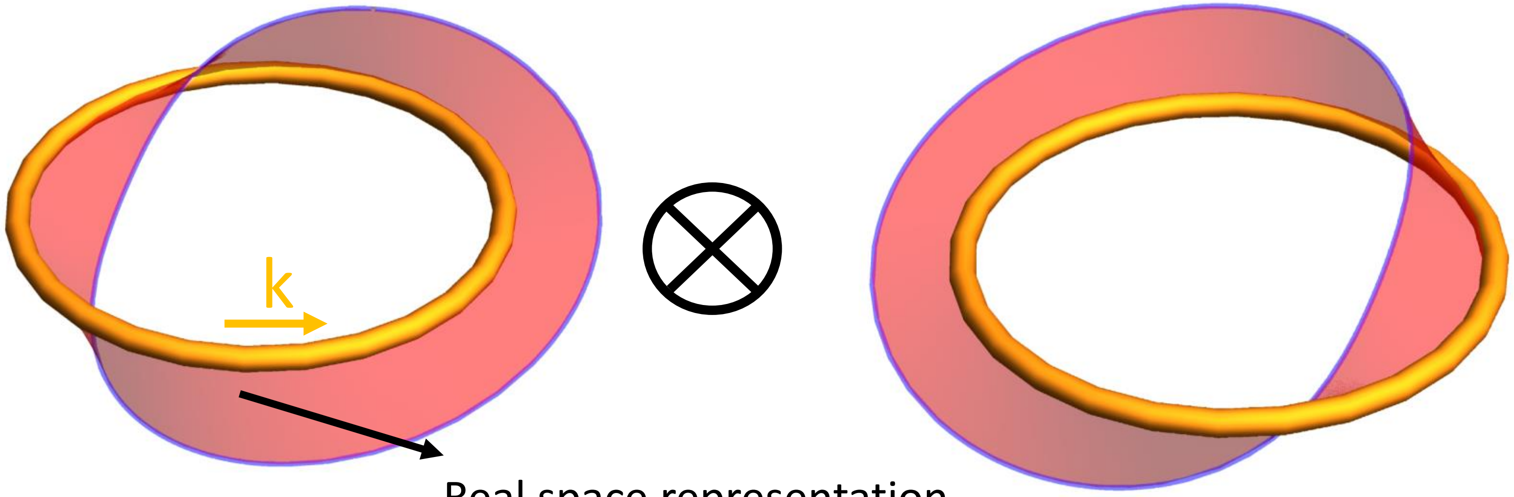
$\mu = 0$



Free Majorana operator at edge

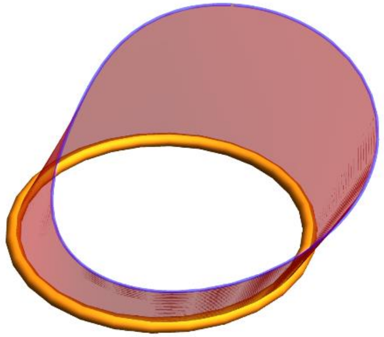
The „knot“ of 1D Majorana zero modes

$$H=\frac{1}{2\pi}\sum_k\left(\mu+\epsilon\tilde{c}_k^\dagger\tilde{c}_k+\lambda\tilde{c}_k^\dagger\tilde{c}_{-k}^\dagger+\lambda^*\tilde{c}_{-k}\tilde{c}_k\right)$$

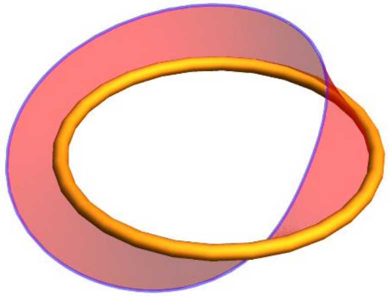


Real space representation
of Hamiltonian in „Pauli-space“

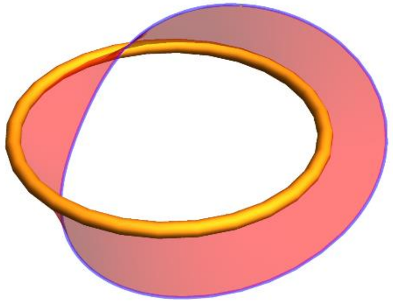
trivial



+ twist

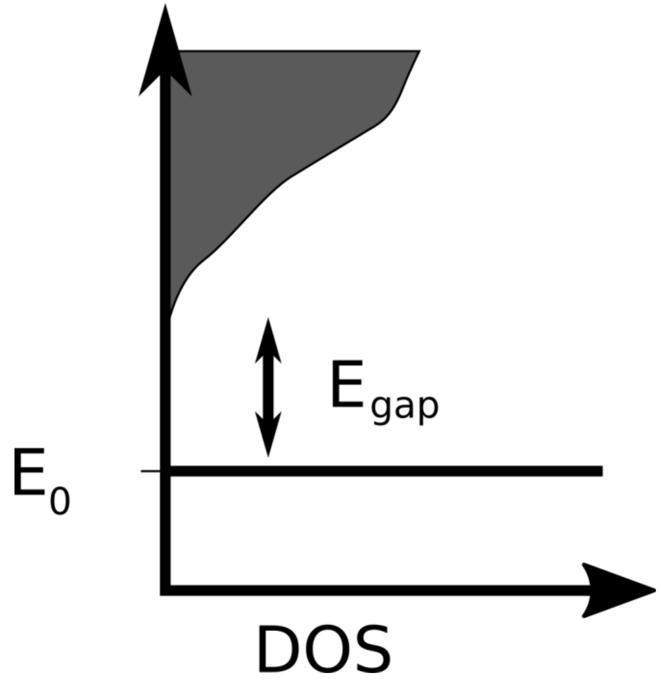


- twist



Topological protection

The superconducting gap is the energy scale that protects the Majorana zero mode



http://www.escapefromcubiclenation.com/broken-coffee-cup/

Dephasing and leakage dynamics of noisy Majorana-based qubits: Topological versus Andreev

Ryan V. Mishmash,[1,2,3,4] Bela Bauer,[5] Felix von Oppen,[6] and Jason Alicea[3,4]

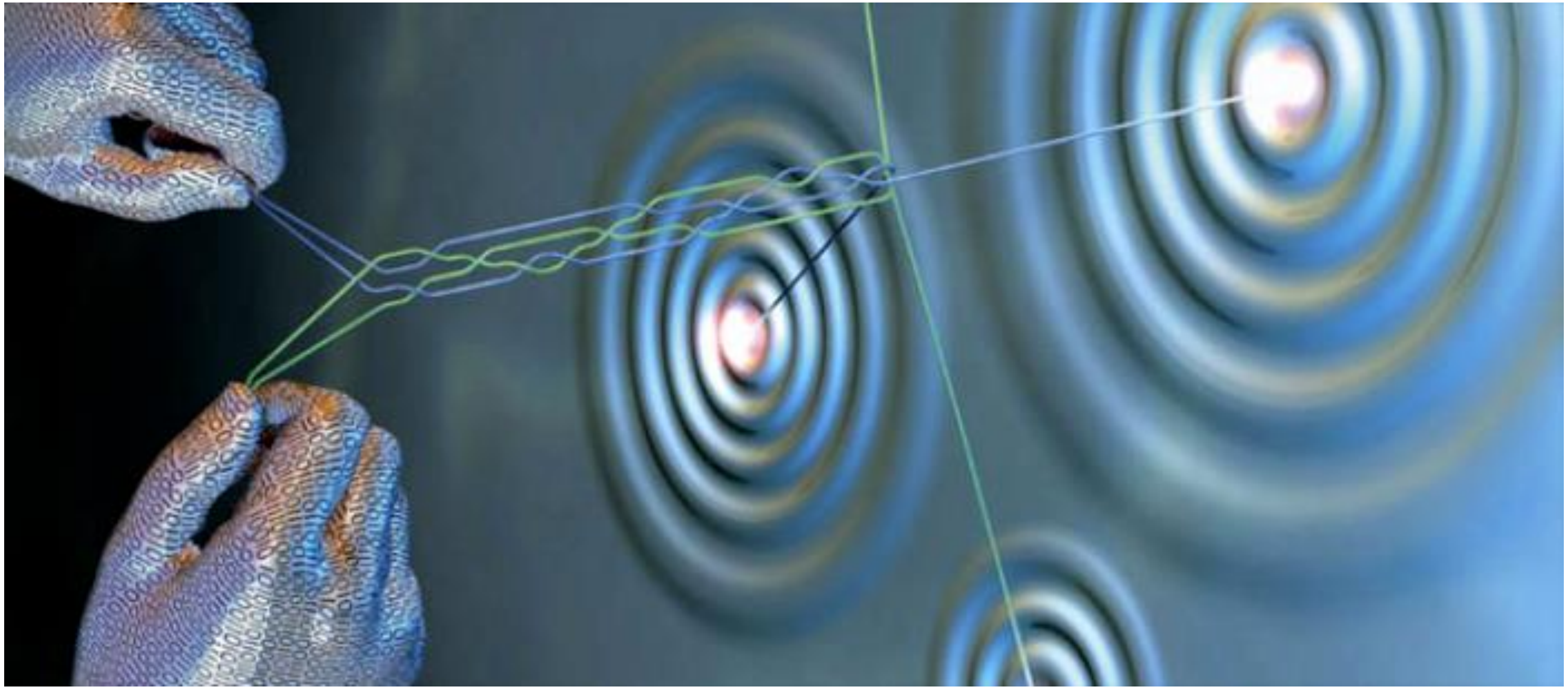
Arxiv:1911.02582 (2019)

Failure of protection of Majorana based qubits against decoherence

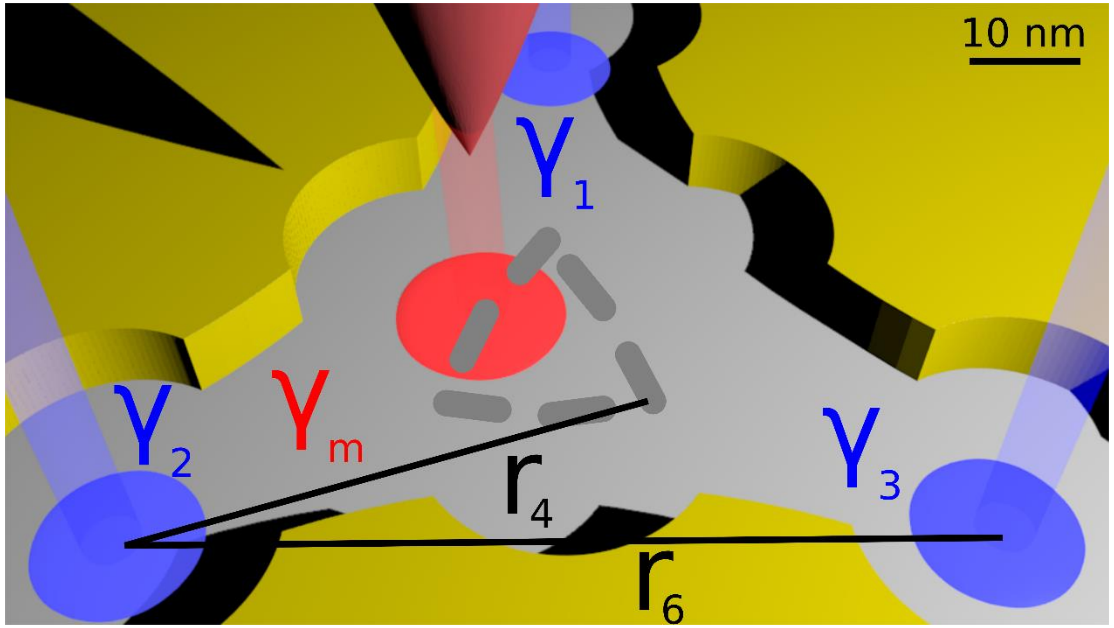
Jan Carl Budich[1], Stefan Walter[1], and Björn Trauzettel[1]

Arxiv:1111.1734 (2012)

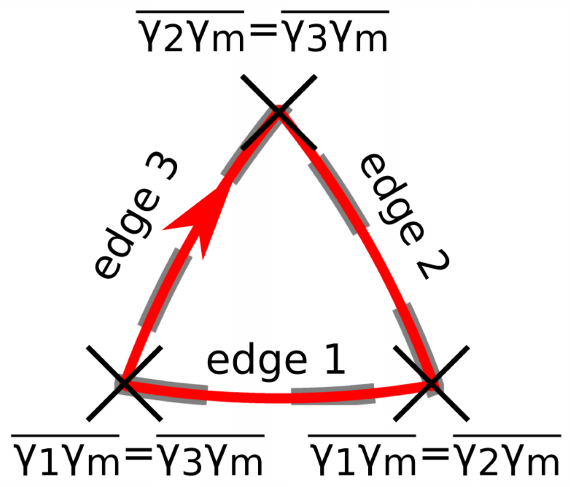
(II) Majorana braiding



Simplifying Majorana braiding

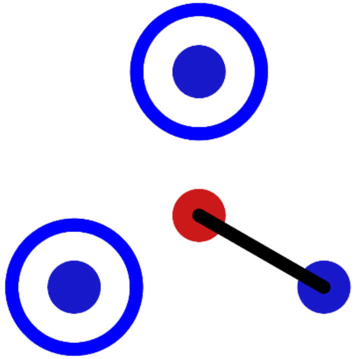


Posske, Qiu, Thorwart arxiv (2019)



$$r_j = \pi\left(j - 3/4\right)/k_F$$

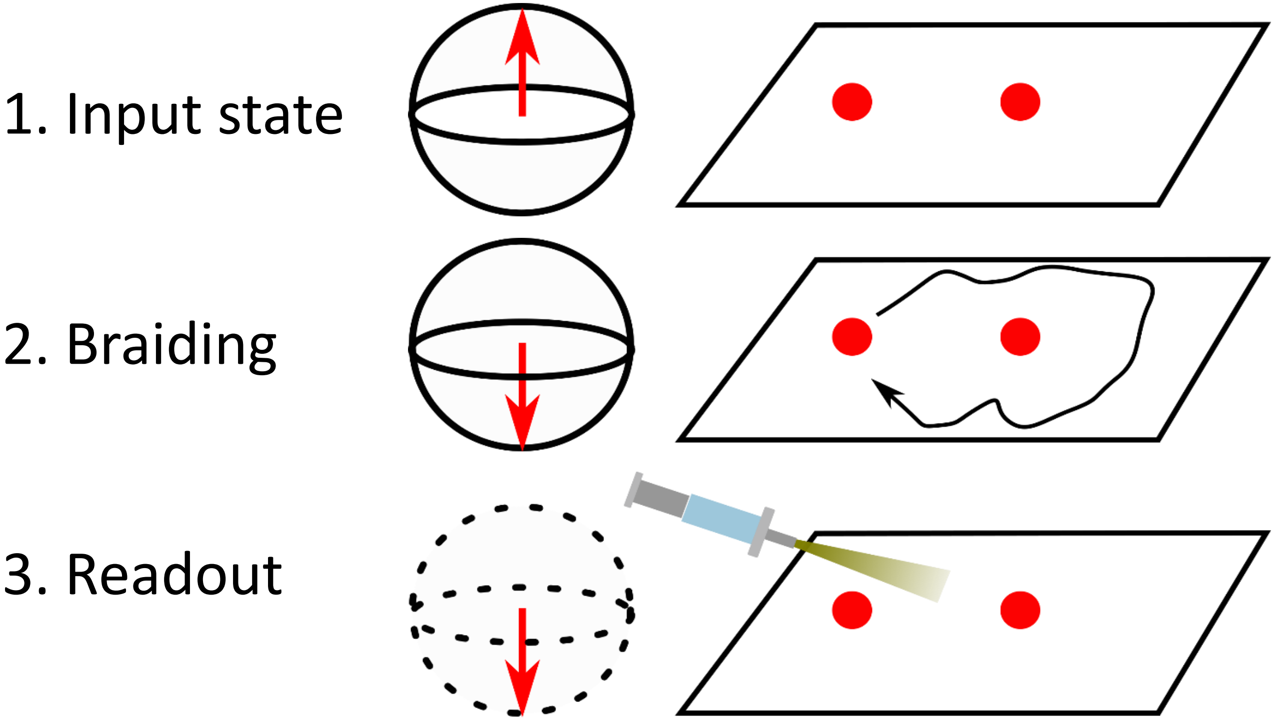
Ching-Kai Qiu
KITS, UCAS, Beijing



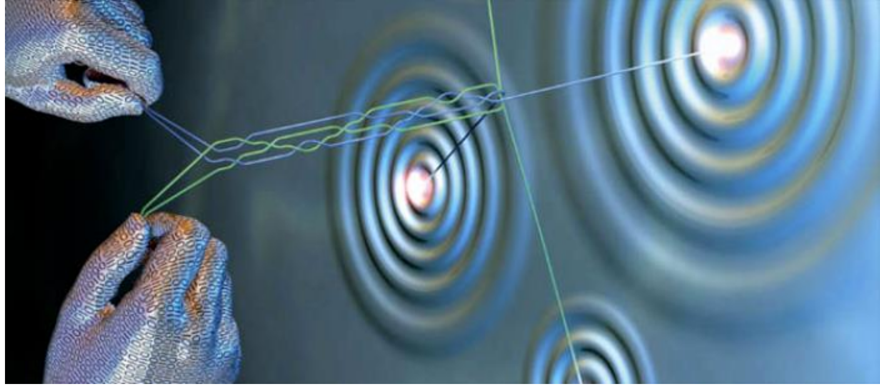
Only slightly move Majoranas in **finite-time**
Geometry where Majoranas mostly decouple

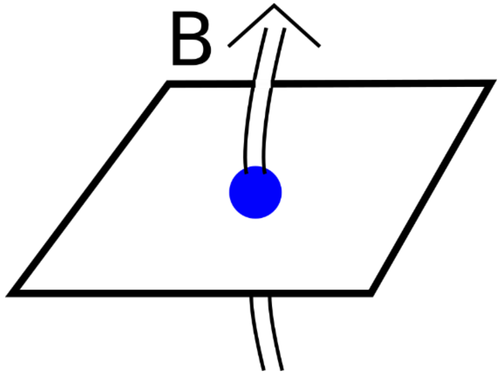
Topological quantum computing by Majorana braiding

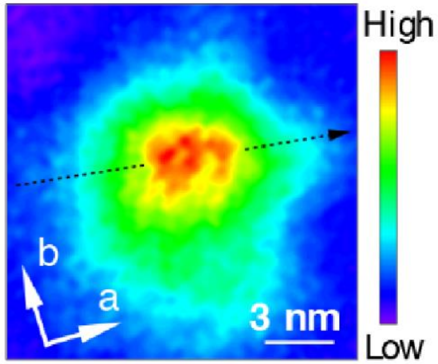
- Quantum computing: qualitatively faster
- Microsoft Station Q Research



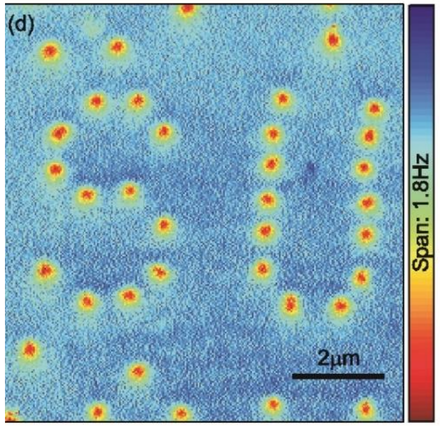
Braiding Majorana modes pinned to vortices



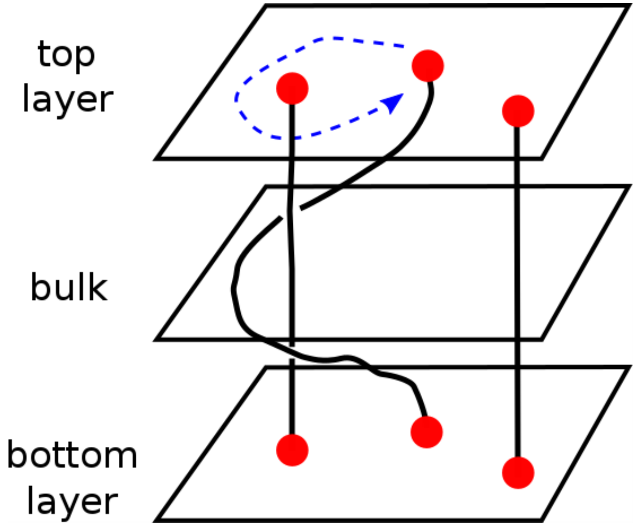




Wang et al.,
Science (2018)

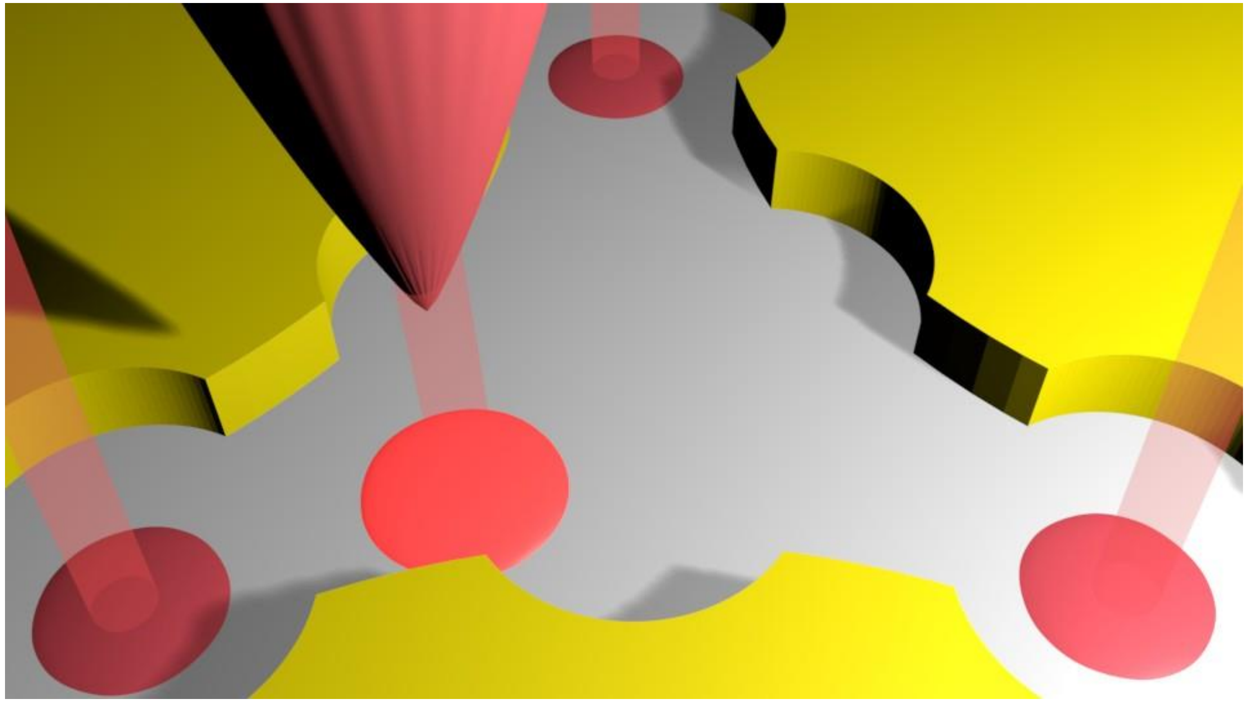


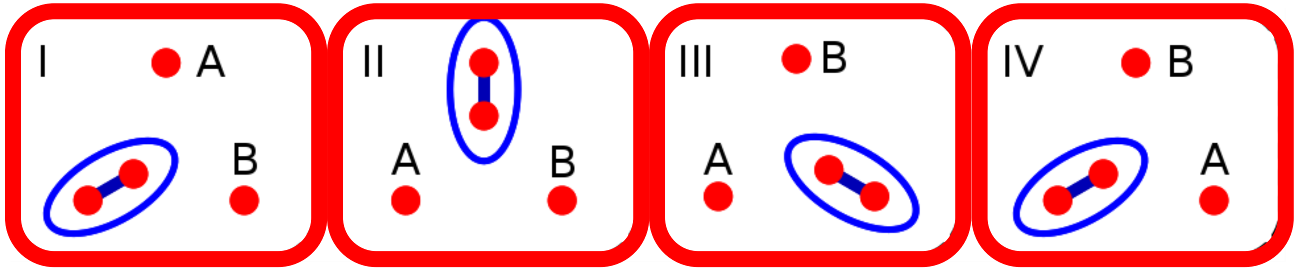
Ge et al.,
Nat. Comm. (2016)



- Braiding: Slowly moving Majorana modes around each other
- Problems:
  - Long braiding times, no coherent quantum control
  - Twisting and relaxation of vortex lines

Vortex Majorana braiding in a finite time

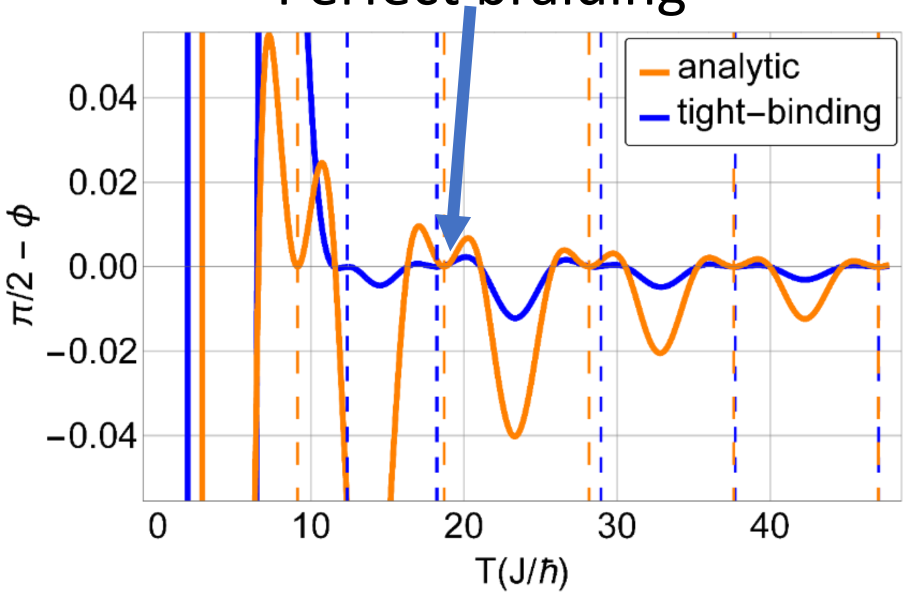




Dynamical control of couplings

Posske, Chiu, Thorwart, arxiv (2019), likely PRR (2020)

Perfect braiding



$$U_{i,j}(t) = e^{-\gamma_i\gamma_j \frac{3\pi t}{2T}} e^{2\gamma_i\gamma_m Jt/\hbar + \gamma_i\gamma_j \frac{3\pi t}{2T}}$$

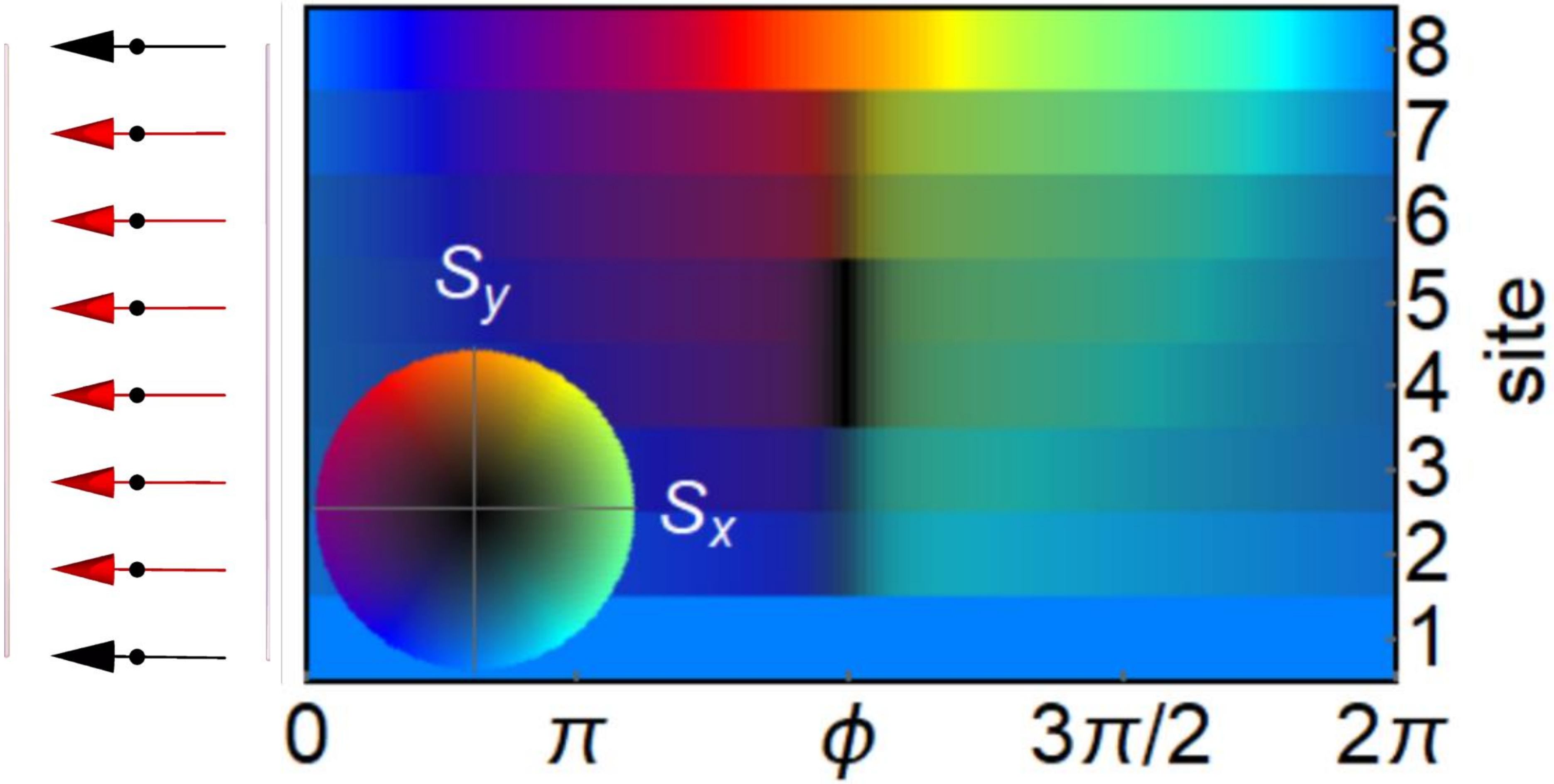
- First analytic solution of finite-time Majorana braiding
- Proved topological robustness
- Numerically confirmed with realistic parameters

Winding-up quantum spin spirals

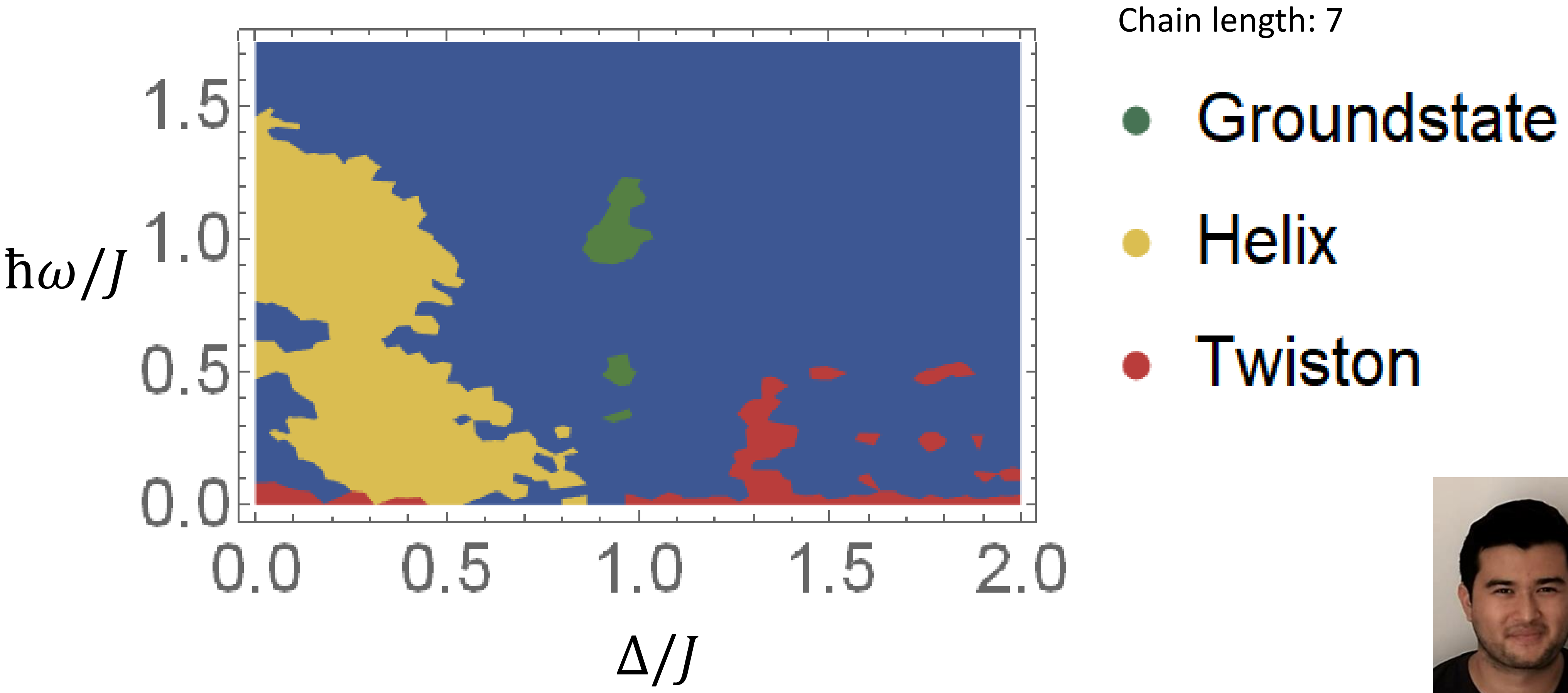
Quantum expectation values

Quantum spin slippage

$|\mathrm{Singlet}\rangle \propto |\downarrow\rangle - |\uparrow\rangle$

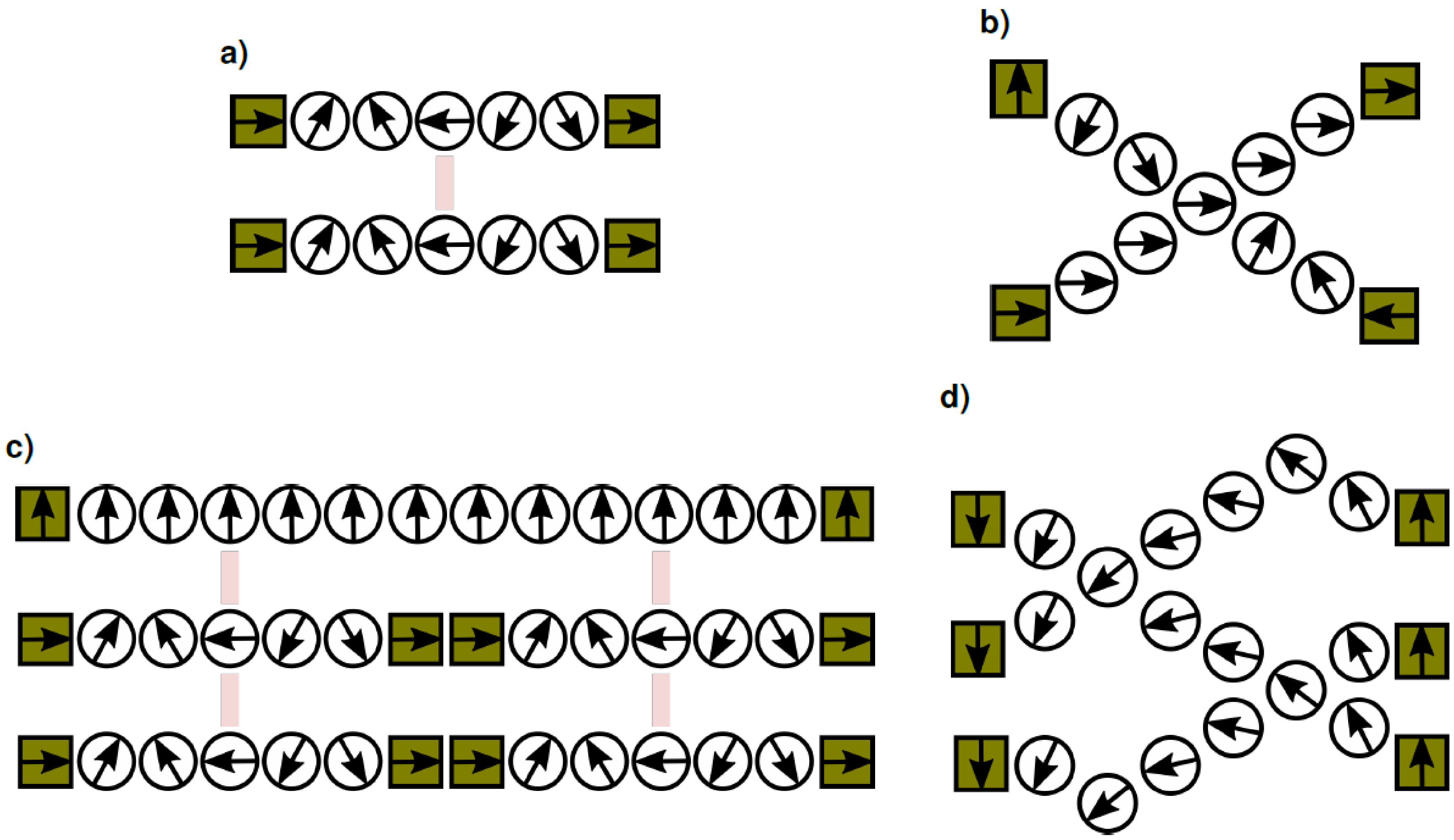


Dynamical creation of helices



Aydin Bittner

Spin-chain-based quantum computing



Majorana the game ©